

The Impact of a Corporate Leniency Program on Antitrust Enforcement and Cartelization*

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Abstract

To explore the efficacy of programs designed to fight cartels, a Markov process is constructed which models the stochastic formation and demise of cartels. Cartels are born when given the opportunity and market conditions are right, while cartels die because of internal collapse or they are caught and convicted by the antitrust authority. The likelihood that a cartel, once identified, is convicted depends inversely on the caseload of the antitrust authority. The antitrust authority also optimally chooses an enforcement policy in terms of how many cases it prosecutes. Holding the enforcement policy of the antitrust authority fixed, a leniency program is shown to lower the frequency of cartels. However, the additional caseload provided by the leniency program induces the antitrust authority to be less aggressive prosecuting cases outside of the program. It is then possible that the cartel rate is higher when there is a leniency program.

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1 Introduction

The 1993 revision of the Corporate Leniency Program of the U.S. Department of Justice's Antitrust Division gives a member of a cartel the opportunity to avoid government penalties if it is the first to fully cooperate and provide evidence. This program is arguably the most significant policy development in the fight against cartels since the Clayton Act instituted private treble damages in 1914. As noted by Antitrust Division officials, the leniency program is the primary generator of cartel cases, and the information provided by those admitted to the program has been instrumental in securing the convictions of other cartel members. Deputy Assistant Attorney General Scott Hammond stated in 2005:

The Antitrust Division's Corporate Leniency Program has been the Division's most effective investigative tool. Cooperation from leniency applicants has cracked more cartels than all other tools at our disposal combined.¹

The widespread usage of the leniency program in the U.S. soon led to the adoption of similar programs in other countries. In 1996, the European Commission (EC) instituted a leniency program and a decade later 24 out of 27 EU members had one. Today, leniency programs span the globe from Canada to the United Kingdom to Japan to South Africa to Brazil. Since 1995, more than 20% of discovered international cartels have been awarded amnesty by at least one antitrust authority (Connor, 2008). The EC provided partial or full leniency in 45 of 50 cartel cases decided during 1998-2007, and leniency lowered average fines per cartel by almost 40% from 199 million to 123 million euros (Veljanovski, 2007). In sum, leniency programs have been widely introduced and utilized throughout the world.

In light of the widespread adoption and usage of leniency programs, a considerable body of scholarly work has developed to understand these programs and assess how they can be better designed; a review of much of this research is provided in Spagnolo (2008). Research is especially valuable since heavy usage of a leniency program need not imply that the program has succeeded in the ultimate goal of reducing the number of cartels. This issue is particularly challenging because the number of cartels is not directly observable. In fact, leniency programs could be utilized, yet be promoting collusion because their terms are too generous.

Starting with the pioneering paper of Motta and Polo (2003), there has been a sequence of theoretical analyses including Spagnolo (2003), Aubert, Kovacic, and Rey (2006), and Harrington (2008). While models and results vary, the overall conclusion is that leniency programs make collusion more difficult.² There is also a growing body of experimental work which similarly provides evidence of the efficacy of leniency

¹Scott D. Hammond, "Cracking Cartels With Leniency Programs," OECD Competition Committee, Paris, France, October 18, 2005.

²There are a variety of effects at work when a leniency program is put in place and some serve to make collusion easier; see Ellis and Wilson (2001) and Chen and Harrington (2007). Generally, these effects net out so that fewer cartels form when there is a leniency program.

programs including most recently Hinloopen and Soetevent (2008) and Bigoni et al (2008). Both studies report that a leniency program reduces cartel formation though Bigoni et al (2008) also find that prices are higher, conditional on a cartel forming. Finally, there are two empirical studies measuring the impact of leniency programs. Using data over 1985-2005 for the United States, Miller (2009) finds evidence that the 1993 revision reduced the latent cartel rate. In contrast, Brenner (2009) does not find evidence that collusion was made more difficult with the European Commission's 1996 Corporate Leniency Program, though his data is for 1990- 2003 and thus does not encompass an important revision in the program in 2002.

While the empirical evidence is mixed and scant at this point, the general conclusion from theoretical and experimental research is that leniency programs are effective in shutting down cartels and deterring cartel formation. However, those findings were derived under a key but problematic assumption that enforcement through non-leniency means is unaffected by the introduction of a leniency program. More specifically, it is assumed that the probability that a cartel is discovered, prosecuted, and convicted - in the absence of a firm coming forward under the leniency program - is unchanged with the adoption of a leniency program. Not only is that assumption almost certain to be violated, but conclusions about the efficacy of a leniency program could significantly change once this probability is made endogenous. Let us argue both points.

With the introduction of a leniency program, the investigation of cases not involving leniency is likely to change and, as a result, this will affect the probability that a cartel is caught and convicted. As an antitrust authority has limited resources, if resources are used to handle leniency cases then fewer resources are available to effectively prosecute non-leniency cases. This doesn't necessarily imply that non-leniency enforcement is weaker, however. If a leniency program is successful in reducing the number of cartels, there will be fewer non-leniency cartel cases, in which case the authority may still have ample resources to effectively prosecute them. Furthermore, an optimizing antitrust authority is likely to adjust its enforcement policy - for example, how it allocates prosecutorial resources across cases - in response to what is occurring with leniency applications. Thus, while we expect the probability that a cartel is caught and convicted to change when a leniency program is put in place, it isn't clear in which direction it will go.

The next point to note is that a change in the likelihood of getting a conviction for a non-leniency case has implications for the efficacy of the leniency process itself. A cartel member will apply for leniency only if it believes that doing so is better than running the risk of being caught and paying full penalties. Thus, the probability of being caught and convicted is integral to inducing firms to apply for leniency. If this probability is very low then no cartel member will use the leniency program, while if the probability is sufficiently high then, under the right circumstances, a cartel member will apply for leniency. The efficacy of a leniency program is then intrinsically tied to how a leniency program affects the probability of being caught and convicted when no firm applies for leniency.

These issues have been recognized in the policy realm as legitimate concerns. In recent years the Directorate General Competition (DG Comp) of the European Commission has been overwhelmed with leniency applications which limits the amount of resources for prosecuting other cases:

DG Competition is now in many ways the victim of its own success; leniency applicants are flowing through the door of its Rue Joseph II offices, and as a result the small Cartel Directorate is overwhelmed with work. ... It is open to question whether a Cartel Directorate consisting of only approximately 60 staff is really sufficient for the Commission to tackle the 50 cartels now on its books.³

Furthermore, the interaction between adoption of a leniency program and enforcement through means other than leniency applications is emphasized in Friederiszick and Maier-Rigaud (2008). Both authors are members of DG Comp and their paper discusses how active DG Comp should be in detecting cartels and more generally being pro-active in initiating cases, given the purported success of the leniency program.

The objective of the current paper is to take all of these considerations into account in order to provide a more comprehensive theoretical examination of the impact of a leniency program on the frequency of collusion. To begin, previous work has explored how a leniency program affects the stability of a single cartel; is collusion made more or less difficult? Our objective is more ambitious in wanting to make statements about the fraction of industries that are cartelized. For this purpose, a population of industries is modelled, each of which decides whether to form a cartel. The opportunity to form a cartel is stochastic and, once that opportunity presents itself, a cartel forms if and only if it is incentive compatible. At the same time that a cartel is stochastically born, it can stochastically die because it is hit by market conditions that cause internal collapse or because it is discovered and then successfully prosecuted by the authorities. After its death, a cartel can stochastically reconstitute itself in the future. We have then constructed a Markov process over the cartel status of a population of industries. The cartel rate is the steady-state percentage of industries which are cartelized.

Into this environment, a corporate leniency program is introduced so that a cartel member has the option of going to the authorities and receiving a reduction in fines. Furthermore, the antitrust authority faces an implicit resource constraint in that the more cases it pursues, the lower is the probability of gaining a conviction for a case which lacks an informant through the leniency program. Finally, we allow the antitrust authority to optimally adjust its enforcement policy with respect to the investigation of non-leniency cases. In doing so, we depart from the standard assumption in the literature on antitrust policy - which assumes an antitrust authority maximizes social welfare⁴ - and instead specify an objective motivated by career

³Alan Riley, "Developing Criminal Cartel Law: Dealing with the Growing Pains," *Competition Law Review*, 4 (2007), 1-6; pp. 1-2.

⁴See, for example, Besanko and Spulber (1989), Motta and Polo (2003), and Harrington (2008).

concerns.

To summarize our main findings, first we show that, holding fixed the non-leniency enforcement policy of the antitrust authority, a leniency program has the desired effect of reducing the cartel rate. This result is consistent with previous theoretical and experimental work. Thus, encompassing a resource constraint for the antitrust authority does not, by itself, alter the conclusion that leniency programs are effective in deterring cartel formation. Second, when the antitrust authority is allowed to adjust its enforcement policy, we find it becomes less aggressive in pursuing cases which do not involve an applicant to the leniency program. Third, after allowing the antitrust authority to adjust its enforcement policy, the introduction of a leniency program can either lower or raise the frequency of cartels. There are many situations for which the antitrust authority sufficiently reduces its enforcement with respect to non-leniency cases that the fraction of industries that are cartelized is actually higher after the introduction of a leniency program. This higher cartel rate is driven by the differential impact of the leniency program across heterogeneous cartels. For industries that, due to market conditions, produce relatively unstable cartels, a leniency program results in cartels no longer forming. But for industries that produce relatively stable cartels, a leniency program does not deter cartel formation and, in fact, results in longer-lived cartels because the investigation of non-leniency cases is less aggressive. A leniency program then results in fewer cartels forming but those that do form could last longer. Of particular relevance to recent history is that we find a leniency program can be extensively used, yet the latent cartel rate is higher.

In the next section, the model is presented. In Section 3, we describe how the equilibrium cartel rate and optimal antitrust policy are determined. The main results are provided in Section 4, while Section 5 concludes.

2 Model

Our modelling strategy is to build a birth and death Markov process for cartels in order to generate an average cartel rate for a population of industries, and to then assess how the introduction of a leniency program influences the frequency of cartels. We build upon the birth and death process developed in Harrington and Chang (2008) by allowing for a leniency program and, most crucially, endogenizing the probability that a cartel ultimately pays penalties. This latter modification is performed by modelling how the antitrust authority's caseload influences the efficacy of their investigations and also how the antitrust authority decides how aggressive to be in investigating cases not brought under the leniency program.⁵

2.1 Industry Environment

Firm behavior is modelled using a modification of a Prisoners' Dilemma formulation. Firms simultaneously decide whether to *collude* or *compete*. Prior to making that

⁵The Harrington-Chang birth and death process for cartels is estimated in Hyytinen, Steen, and Toivanen (2010) who use data from the Finnish cartel registry.

choice, firms observe a stochastic realization of the market's profitability that is summarized by the variable $\pi \geq 0$.⁶ If all firms choose *collude* then each firm earns π , while if all choose *compete* then each earns $\alpha\pi$ where $\alpha \in [0, 1)$. $1 - \alpha$ then measures the competitiveness of the non-collusive environment. π has a continuously differentiable cdf $H : [\underline{\pi}, \bar{\pi}] \rightarrow [0, 1]$ where $0 < \underline{\pi} < \bar{\pi}$. $h(\cdot)$ denotes the associated density function and let $\mu \equiv \int \pi h(\pi) d\pi$ denote its finite mean. If all other firms choose *collude*, the profit a firm earns by deviating - choosing *compete* - is $\eta\pi$ where $\eta > 1$. This information is summarized in the table below. Note that the Bertrand price game is represented by $(\alpha, \eta) = (0, n)$ where n is the number of firms. The Cournot quantity game with linear demand and cost functions in which firms collude at the joint profit maximum is represented by $(\alpha, \eta) = \left(\frac{4n}{(n+1)^2}, \frac{(n+1)^2}{4n}\right)$.⁷

Own action	All other firms' actions	Own profit
<i>collude</i>	<i>collude</i>	π
<i>compete</i>	<i>collude</i>	$\eta\pi$
<i>compete</i>	<i>compete</i>	$\alpha\pi$

Firms interact in an infinite horizon setting where $\delta \in (0, 1)$ is the common discount factor. It is not a repeated game because, as explained later, each industry is in one of two states: it can be a cartel or not. If *firms are a cartel* then they will effectively collude only when it is incentive compatible. More specifically, if firms are cartelized then they simultaneously choose between *collude* and *compete*, and, at the same time, whether or not to apply to the corporate leniency program. Details on the description of the leniency program are provided later. If it is incentive compatible for all firms to choose *collude* then each earns π . If instead a firm prefers *compete* when all other firms choose *collude* then collusion is not incentive compatible (that is, it is not part of the subgame perfect equilibrium for the infinite horizon game) and each firm earns $\alpha\pi$. In that case, collusion is not achieved. If *firms are not a cartel* then each firm earns $\alpha\pi$ as, according to equilibrium, they all choose *compete*.

At the end of the period, there is the random event whereby the antitrust authority (AA) may pursue an investigation; this can only occur if firms colluded in the current or previous period and no firm applied for leniency.⁸ Let $\sigma \in [0, 1)$ denote the probability that firms are discovered, prosecuted, and convicted (below, we will endogenize σ though, from the perspective of an individual industry, it is fixed). In that event, each firm incurs a penalty of $\frac{F}{1-\delta}$ (so that F is the per-period penalty).

It is desirable to allow F to depend on the extent of collusion. Given there is only one level of collusion in the model, the "extent of collusion" necessarily refers

⁶The informational setting is as in Rotemberg and Saloner (1986).

⁷We have only specified a firm's profit when all firms choose *compete*, all firms choose *collude*, and it chooses *compete* and all others firm choose *collude*. We must also assume that *compete* strictly dominates *collude* for the stage game. It is unnecessary to provide any further specification.

⁸To allow it to depend on collusion farther back in time would require introducing another state variable that would unnecessarily complicate the analysis. Having it depend on collusion in the previous period will simplify some of the expressions and, furthermore, it seems quite reasonable that detection can occur, to a limited extent, after the fact.

to the number of periods that firms had colluded. A proper accounting of that effect would require that each cartel have a state variable which is the length of collusion; this would seriously complicate the analysis. As an approximation, we instead assume that the penalty is proportional to the average increase in profit from being cartelized (rather than the realized increase in profit). If Y denotes the expected per period profit from being in the "cartel state" then $F = \gamma(Y - \alpha\mu)$ where $\gamma > 0$ and $\alpha\mu$ is average non-collusive profit. This specification avoids the need for state variables but still allows the penalty to be sensitive to the (average) extent of collusion.⁹

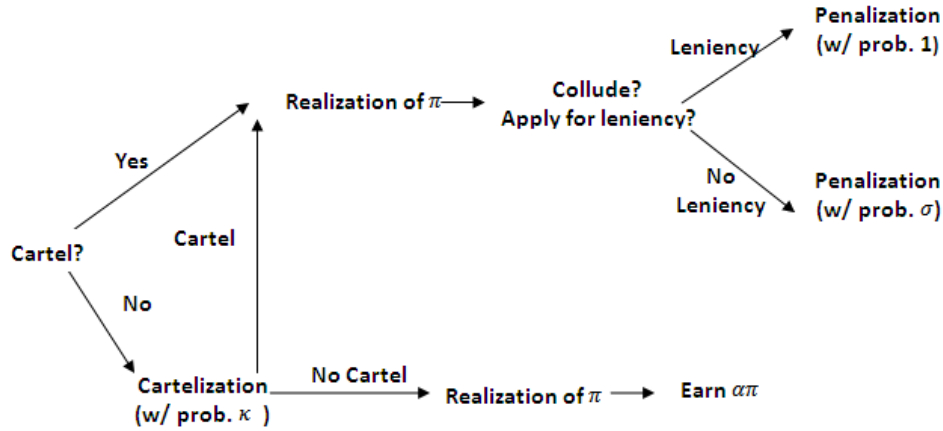
In addition to being discovered by the authorities, a cartel can be uncovered because one of its members comes forth under the corporate leniency policy. Suppose a cartel is in place. If a single firm applies for leniency then all firms are convicted for sure and the firm that applied receives a per period penalty of θF where $\theta \in [0, 1]$, while the other cartel members each pay F . If all firms simultaneously apply for leniency then each firm pays a penalty of ωF where $\omega \in (\theta, 1)$. For example, if only one firm can receive leniency and each firm has an equal probability of being first in the door then $\omega = \frac{n-1+\theta}{n}$ when there are n cartel members. It is sufficient for the ensuing analysis that we specify the leniency program when either one firm applies or all firms apply. Also, leniency is not awarded to firms that apply after another firm has done so. However, we do not think qualitative results would change if we allowed more than one firm to receive leniency.

From the perspective of firms, antitrust policy is summarized by the four-tuple $(\sigma, \gamma, \theta, \omega)$ which are, respectively, the probability of paying penalties (in the absence of any firm using the leniency program), the penalty multiple, the leniency parameter when only one firm applies (where $1 - \theta$ is the proportion of fines waived), and the leniency parameter when all firms apply (where $1 - \omega$ is the proportion of fines waived). As there are assumed to be only corporate penalties, our model is better suited for non-U.S. jurisdictions, such as the European Union, which lack individual penalties.

Next, let us describe how an industry's cartel status evolves. Suppose it enters the period cartelized. The industry will exit the period still being a cartel if: 1) all firms chose *collude* (which requires that collusion be incentive compatible); 2) no firm applied for leniency; and 3) the AA did not discover and convict the firms of collusion. Otherwise, the cartel collapses and firms revert to the "no cartel" state. If instead the industry entered the period in the "no cartel" state then with probability $\kappa \in (0, 1)$ firms cartelize. For that cartel to still be around at the end of the period, conditions (1)-(3) above must be satisfied. Note that whenever a cartel is shutdown - whether due to internal collapse, applying to the leniency program, or having been successfully prosecuted - the industry may re-cartelize in the future. Specifically, it has an opportunity to do so with probability κ in each period that it is not currently

⁹A more standard assumption in the literature is to assume F is fixed which is certainly simpler but less realistic than our specification. All qualitative results hold when F is fixed.

colluding.¹⁰ The timing of events is summarized in the figure below.



In modelling a population of industries, it is compelling to allow industries to vary in terms of cartel stability. For this purpose, industries are assumed to differ in terms of the parameter η . If one takes this assumption literally, it can be motivated by heterogeneity in the elasticity of firm demand or the number of firms (as with the Bertrand price game). Our intent is not to be literal but rather to think of this as a parsimonious way in which to encompass industry heterogeneity. Let the cdf on industry types be represented by the continuously differentiable function $G : [\underline{\eta}, \bar{\eta}] \rightarrow [0, 1]$ where $1 < \underline{\eta} < \bar{\eta}$. $g(\cdot)$ denotes the associated density function. The appeal of η is that it is a parameter which influences the frequency of collusion but does not directly affect the value of the firm's profit stream since, in equilibrium, firms do not cheat; this property makes for an easier analysis.

2.2 Antitrust Enforcement Technology

We now turn to describing how the caseload affects the probability of the AA succeeding with an investigation. In Section 3.4, the objective of the AA is discussed. Recall that σ is the probability that a cartel pays penalties when no member has applied for leniency. It is the compounding of three events: 1) the cartel is discovered by the AA; 2) the AA decides to investigate the cartel; and 3) the AA is successful in its investigation (where success means that penalties are levied, either through a plea agreement or a prosecution and conviction, though we will often refer to this last event as "conviction"). The initial discovery of a cartel is presumed to be exogenous and to come from customers, uninvolved employees, the accidental discovery of evidence through a proposed merger, and so forth. q denotes the probability of discovery and is a parameter throughout the paper. What the AA controls is how many cases

¹⁰ Alternatively, one could imagine having two distinct probabilities - one to reconstitute collusion after a firm cheated (the probability of moving from the punishment to the cooperative phase) and another to reform the cartel after having been convicted. For purposes of parsimony, those two probabilities are assumed to be the same.

to take on which is represented by r which is the fraction of reported cases that the AA chooses to investigate. Finally, of those cases discovered and investigated, the AA is successful in a fraction s of them where s will depend on the AA's caseload.¹¹

The AA is faced with a resource constraint: the more cases it takes on, the fewer resources are applied to each case and the lower is the probability of winning any individual case. More specifically, we assume

$$s = p(\lambda L + R) \text{ where } \lambda \in [0, 1].$$

L is the number (or mass) of leniency cases, R is the number of non-lenieny cases, and s is the probability of winning one of the R cases, where it is assumed leniency cases are won for sure. $\lambda \in [0, 1]$ seems natural as leniency cases ought to take up fewer resources than those cases lacking an informant. p is assumed to be a decreasing function so that a bigger caseload means a lower probability of winning a non-lenieny case. In sum, the probability that a cartel pays penalties is

$$\sigma = q \times r \times s = q \times r \times p(\lambda L + R).$$

It is endogenous as the AA chooses r and, in addition, s is determined by the number of leniency and non-lenieny cases which depends on the number of cartels.

We chose a fairly flexible functional form for how the caseload relates to the probability that the AA is successful in getting penalties out of the cartel:

$$p(\lambda L + R) = \frac{\tau}{\xi + v(\lambda L + R)^\rho}, \text{ where } v > 0, \rho \geq 1, \tau \in (0, 1], \xi \geq \tau.$$

Note that

$$p(0) = \frac{\tau}{\xi}, \lim_{\lambda L + R \rightarrow +\infty} p(\lambda L + R) = 0.$$

As long as $\rho > 1$, $p(\lambda L + R)$ is initially concave and then convex with an inflection point at

$$\lambda L + R = \left(\frac{\xi(\rho - 1)}{v(\rho + 1)} \right)^{1/\rho}.$$

For all values of ξ and v considered in this paper, the inflection point is increasing in ρ , which means that the range over which it is concave is larger when ρ is higher.¹²

In concluding, let us discuss the assumption that the AA faces a resource constraint in the sense that, implicitly, resources per case decline with the number of

¹¹In a richer model, we could allow for heterogeneity across cartel cases in terms of the perceived difficulty of gaining a conviction; that is, s is cartel-specific. The AA would then decide when a case is too difficult to prosecute.

¹²It should be noted that Motta and Polo (2003) do allow for optimal enforcement expenditure by modelling a trade-off between monitoring and prosecution. They endow an antitrust authority with a fixed amount of resources that can be allocated between finding suspected episodes of collusion and prosecuting the cases that are found or, in the language of our model, between raising q and lowering s (assuming $r = 1$). However, the model is very different from ours - for example, they do not consider a population of industries and do not solve for the steady-state frequency of cartels - and it does not address the questions we are raising here.

cases as reflected in the assumption that the probability of any investigation being successful is decreasing in the size of the caseload. In practice, an AA can move around resources to handle additional cartel activity by, for example, shifting lawyers and economists from merger cases to cartel cases. However, there is a rising opportunity cost in doing so and that ought to imply that resources per cartel case will decline with the number of cartel cases. Of course, AA officials can lobby their superiors (either higher level bureaucrats or elected officials) for a bigger budget but, at least in the U.S., the reality is that the AA's budget does not scale up with its caseload. While the budget of the Antitrust Division of the U.S. Department of Justice is increasing in GDP (Kwoka, 1999), DOJ antitrust case activity is actually countercyclical (Ghosal and Gallo, 2001) which substantiates our assumption that resources per case is declining in the number of cases.

3 Equilibrium Cartel Rate

In this section, we describe the equilibrium frequency with which industries are cartelized. Prior to getting into the details, let us provide a brief overview.

1. Taking as given σ (the per period probability that a cartel pays penalties when the leniency program is not used), we first solve for equilibrium collusive behavior. For a type- η industry, this entails solving for the set of market conditions (values for π) such that collusion is incentive compatible.
2. With step 1 completed, we can then define the Markov process on cartel creation and destruction and solve for the stationary distribution of industries in terms of their cartel status, for each industry type η . By aggregating over all industry types, the equilibrium cartel rate, $C(\sigma)$, is derived, given σ .
3. Next we derive the equilibrium success rate, s^* . The probability that the AA's investigation is successful, $p(\lambda L + R)$, depends on the mass of leniency cases, L , and the mass of non-lenieny cases, R . Note that L and R are both influenced by how many cartels there are, $C(\sigma)$. s^* is then a fixed point: $s^* = p(\lambda L(C(qrs^*)) + R(C(qrs^*)))$, where recall that $\sigma = qrs$. In other words, the probability that firms assign to being caught, investigated, and convicted determines the cartel rate, and the cartel rate determines the number of cases handled by the AA and thus the probability that they are able to get a conviction on a case.
4. The final step is to specify an objective for the AA and then solve for the optimal value for r , which is the fraction of non-lenieny cases that it pursues.

3.1 Cartel Formation and Collusive Value

A collusive strategy for a type- η industry entails colluding when π is sufficiently low and not colluding otherwise. The logic is as in Rotemberg and Saloner (1986). When π is high, the incentive to deviate is strong because a firm increases current

profit by $(\eta - 1)\pi$. At the same time, the future payoff is independent of the current realization of π , given that π is *iid*. Since the payoff to cheating is increasing in π while the future payoff is independent of π , the incentive compatibility of collusion is more problematic when π is higher.

Suppose firms are able to collude for at least some realizations of π , and let W^o and Y^o denote the payoff when the industry is not cartelized and is cartelized, respectively. If not cartelized then, with probability κ , firms have an opportunity to cartelize with resulting payoff Y^o . With probability $1 - \kappa$, firms do not have such an opportunity and continue to compete. In that case, each firm earns current expected profit of $\alpha\mu$ and a future value of W^o . Thus, the payoff when not colluding is defined recursively by:

$$W^o = (1 - \kappa)(\alpha\mu + \delta W^o) + \kappa Y^o. \quad (1)$$

As it'll be easier to work with re-scaled payoffs, define:

$$W \equiv (1 - \delta)W^o, \quad Y \equiv (1 - \delta)Y^o.$$

Multiplying both sides of (1) by $1 - \delta$ and re-arranging yields:

$$W = \frac{(1 - \kappa)(1 - \delta)\alpha\mu + \kappa Y}{1 - \delta(1 - \kappa)}$$

Also note that the incremental value to being in the cartelized state is:

$$Y - W = Y - \left(\frac{(1 - \kappa)(1 - \delta)\alpha\mu - \kappa Y}{1 - \delta(1 - \kappa)} \right) = \frac{(1 - \kappa)(1 - \delta)(Y - \alpha\mu)}{1 - \delta(1 - \kappa)}. \quad (2)$$

Suppose firms are cartelized and π is realized. When a firm decides whether to collude or cheat, it decides at the same time whether to apply for leniency. If it decides to collude, it is clearly not optimal to apply for leniency since the cartel is going to be shut down by the authorities in which case the firm ought to maximize current profit by cheating. The more relevant issue is whether it should apply for leniency if it decides to cheat. The incentive compatibility constraint (ICC) is:

$$(1 - \delta)\pi + \delta[(1 - \sigma)Y + \sigma(W - \gamma(Y - \alpha\mu))] \geq (1 - \delta)\eta\pi + \delta[W - \min\{\sigma, \theta\}\gamma(Y - \alpha\mu)]. \quad (3)$$

Examining the lhs expression, if it colludes then it earns current profit of π (given all other firms are colluding). With probability $1 - \sigma$, the cartel is not shut down by the AA and, given the industry is in the cartel state, the future payoff is Y . With probability σ , the cartel is caught and convicted by the AA - which means a penalty of $\gamma(Y - \alpha\mu)$ - and since the industry is no longer cartelized, the future payoff is W . Turning to the rhs expression, the current profit from cheating is $\eta\pi$. Since this defection causes the cartel to collapse, the future payoff is W . There is still a chance of being caught and convicted. A deviating firm will apply for leniency iff the penalty from doing so is less than the expected penalty from not doing so (and recall that the other firms are colluding and thus do not apply for leniency):

$\theta\gamma(Y - \alpha\mu) < \sigma\gamma(Y - \alpha\mu)$ or $\theta < \sigma$. Given optimal use of the leniency program, the deviating firm's expected penalty is then $\min\{\sigma, \theta\}\gamma(Y - \alpha\mu)$. Re-arranging (3) and using (2), the ICC can be presented as:

$$\pi \leq \frac{\left(\frac{\delta(1-\sigma)(1-\kappa)(1-\delta)(Y-\alpha\mu)}{1-\delta(1-\kappa)}\right) - \delta[\sigma - \min\{\sigma, \theta\}]\gamma(Y - \alpha\mu)}{(1-\delta)(\eta-1)} \equiv \phi(Y, \sigma, \eta). \quad (4)$$

Collusion is incentive compatible iff the current market condition is sufficiently low.

In deriving an expression for the value to colluding, we need to discuss usage of the leniency program in equilibrium. Firms do not use it when market conditions result in the cartel being stable but may use it when the cartel collapses. As the continuation payoff is W regardless of whether leniency is used, a firm applies for leniency iff it reduces the expected penalty. First note that an equilibrium either has no firms applying for leniency or all firms doing so because if at least one firm applies then another firm can lower its expected penalty from F to ωF by also doing so. This has the implication that it is always an equilibrium for all firms to apply for leniency. Furthermore, it is the unique equilibrium when $\theta < \sigma$. To see why, suppose all firms were not to apply for leniency. A firm would then lower its penalty from σF to θF by applying. When instead $\sigma \leq \theta$, there is also an equilibrium in which no firm goes for leniency as to do so would increase its expected penalty from σF to θF . Using the selection criterion of Pareto dominance, we will assume that, upon internal collapse of the cartel, no firms apply when $\sigma \leq \theta$ and all firms apply when $\theta < \sigma$.

The expected payoff to being cartelized, $\psi(Y, \sigma, \eta)$, is then recursively defined by:

$$\psi(Y, \sigma, \eta) = \begin{cases} \int_{\underline{\pi}}^{\phi(Y, \sigma, \eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma(W - \gamma(Y - \alpha\mu))]\} h(\pi) d\pi & \text{if } \sigma \leq \theta \\ + \int_{\phi(Y, \sigma, \eta)}^{\bar{\pi}} [(1-\delta)\alpha\pi + \delta W - \delta\sigma\gamma(Y - \alpha\mu)] h(\pi) d\pi & \\ \int_{\underline{\pi}}^{\phi(Y, \sigma, \eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma(W - \gamma(Y - \alpha\mu))]\} h(\pi) d\pi & \text{if } \theta < \sigma \\ + \int_{\phi(Y, \sigma, \eta)}^{\bar{\pi}} [(1-\delta)\alpha\pi + \delta W - \delta\omega\gamma(Y - \alpha\mu)] h(\pi) d\pi & \end{cases}$$

To understand this expression, first consider when $\sigma \leq \theta$, in which case leniency is never used. If $\pi \in [\underline{\pi}, \phi(Y, \sigma, \eta)]$ then collusion is incentive compatible; each firm earns current profit of π and an expected future payoff of $(1-\sigma)Y + \sigma(W - \gamma(Y - \alpha\mu))$. If instead $\pi \in (\phi(Y, \sigma, \eta), \bar{\pi}]$ then collusion is not incentive compatible, so each firm earns current profit of $\alpha\pi$ and an expected future payoff of $W - \sigma\gamma(Y - \alpha\mu)$. The expression when $\theta < \sigma$ differs only with respect to when collusion breaks down in which case the future payoff is $W - \omega\gamma(Y - \alpha\mu)$ as all firms apply for leniency.

Note that if market conditions are sufficiently strong - that is, $\pi > \phi(Y, \sigma, \eta)$ - firms not only do not collude (as it is not incentive compatible) but the cartel breaks down, as reflected in firms having a future payoff of W (less expected penalties) An alternative strategy is to have firms not collude when market conditions are strong but for the cartel to remain in place so that firms collude again as soon as market conditions return to lower levels, in which case the future payoff is Y . The latter equilibrium is more in the spirit of the traditional approach to modelling collusive

behavior in that the degree of collusion adjusts to market conditions rather than cartel breakdown occurring. We do not characterize such an equilibrium because, in practice, cartels do breakdown - it is not simply that firms go to a coordinated punishment - and it is that death process we want our model to generate.¹³

A fixed point to ψ is an equilibrium value for Y given σ . That is, given an anticipated future collusive value Y , the resulting equilibrium behavior - as represented by $\phi(Y, \sigma, \eta)$ - results in firms colluding for market states such that the value to being in a cartel is Y . We then want to solve:

$$Y^*(\sigma, \eta) = \psi(Y^*(\sigma, \eta), \sigma, \eta).$$

As an initial step to exploring the set of fixed points, first note that $\psi(\alpha\mu, \sigma, \eta) = \alpha\mu$. Hence, one fixed point to ψ is the degenerate solution without collusion. If there is a fixed point with collusion - that is, $Y > \alpha\mu$ - then we select the one with the highest Y .

Given $Y^*(\sigma, \eta)$, define

$$\phi^*(\sigma, \eta) \equiv \phi(Y^*(\sigma, \eta), \sigma, \eta),$$

as the maximum profit realization such that a type- η cartel is stable. $\phi^*(\sigma, \eta)$ is a measure of cartel stability since the cartel is stable iff $\pi \leq \phi^*(\sigma, \eta)$ and thus internally collapses with probability $1 - H(\phi^*(\sigma, \eta))$.

Before moving on to allowing for a population of industries and endogenizing σ , it is useful to review the various ways in which a leniency program affects the calculus to form and maintain a cartel. Assume σ is fixed and $\theta < \sigma$ so that firms would potentially want to use the leniency program. Previous work has shown that the introduction of a leniency program has three effects; these effects are present in Harrington (2008). First, it makes cartels less stable by reducing the penalties that a firm receives when it cheats; expected penalties to a deviating firm decline from σF to θF . Referred to as the Deviator Amnesty effect, it tightens the incentive compatibility constraint in (3) and thereby reduces the maximum market state for which collusion is incentive compatible, ϕ^* . Second, the probability of paying penalties is higher because firms in a collapsing cartel will apply for leniency. The probability rises from σ to $\sigma H(\phi^*) + (1 - H(\phi^*))$ where $\sigma H(\phi^*)$ is the probability that a cartel does not collapse but is caught and convicted by the AA and $1 - H(\phi^*)$ is the probability that a cartel internally collapses and firms subsequently apply for leniency. This is the Race to the Courthouse effect. Third, the Cartel Amnesty effect of a leniency program lowers the penalties that cartel members pay in equilibrium. When the cartel collapses and firms apply for leniency, penalties are reduced from F to ωF . This last effect promotes cartel formation. When σ is fixed, Harrington (2008) shows that, generally, these three counter-acting effects net out so that a leniency program makes collusion more difficult. Of course, we are endogenizing σ in the current model by assuming it is lower when the AA's caseload is bigger and also endogenizing the

¹³In a richer model in which firms could choose from an array of prices, we would be fine with having some adjustment of the collusive price to market conditions - rather than always having cartel breakdown - as long as, under some market conditions, the cartel does collapse.

AA's enforcement policy. This introduces a potentially significant feedback effect in that more effective enforcement (either through a leniency program or the value of r) can reduce the cartel rate, which can then make enforcement more effective (by raising the probability of a conviction) which can lower the cartel rate more, and so on.

3.2 Stationary Distribution

Given $\phi^*(\sigma, \eta)$, the stochastic process by which cartels are born and die (either through internal collapse or being shut down by the AA) is characterized in this section. The random events driving this process are the opportunity to cartelize, market conditions, and conviction by the AA. We initially characterize the stationary distribution for type- η industries. The stationary distribution for the entire population of industries is then derived by integrating the type specific distributions over all types.

Consider an arbitrary type- η industry. If it is not cartelized at the end of the preceding period then, by the analysis in Section 3.1, it'll be cartelized at the end of the current period with probability $\kappa(1 - \sigma)H(\phi^*(\sigma, \eta))$. With probability κ it has the opportunity to cartelize, with probability $H(\phi^*(\sigma, \eta))$ the realization of π is such that collusion is incentive compatible, and with probability $1 - \sigma$ it is not caught and convicted by the AA. If instead the industry was cartelized at the end of the previous period, it'll still be cartelized at the end of this period with probability $(1 - \sigma)H(\phi^*(\sigma, \eta))$. Suppose there is a continuum of type- η industries with independent realizations of the stochastic events each period. The task is to characterize the stationary distribution with regards to the frequency of cartels.

Let $\beta(\eta)$ denote the proportion of type- η industries which are not cartelized. The stationary rate of non-cartels is defined by :

$$\begin{aligned} \beta(\eta) = & \beta(\eta) [(1 - \kappa) + \kappa(1 - H(\phi^*)) + \kappa\sigma H(\phi^*)] \\ & + [1 - \beta(\eta)] [(1 - H(\phi^*)) + \sigma H(\phi^*)] \end{aligned} \quad (5)$$

Examining the rhs of (5), a fraction $\beta(\eta)$ of type- η industries were not cartelized in the previous period. Out of those industries, a fraction $1 - \kappa$ will not have the opportunity to cartelize in the current period. A fraction $\kappa(1 - H(\phi^*))$ will have the opportunity but, due to a high realization of π , find it is not incentive compatible to collude, while a fraction $\kappa\sigma H(\phi^*)$ will cartelize and collude but then are discovered by the AA. Of the industries that were colluding in the previous period, which have mass $1 - \beta(\eta)$, a fraction $1 - H(\phi^*)$ will collapse for internal reasons and a fraction $\sigma H(\phi^*)$ will instead be caught by the authorities and thus shut down.

Solving (5) for $\beta(\eta)$:

$$\beta(\eta) = \frac{1 - (1 - \sigma)H(\phi^*)}{1 - (1 - \kappa)(1 - \sigma)H(\phi^*)}. \quad (6)$$

For the stationary distribution, the fraction of cartels among type- η industries is then:

$$C(\sigma, \eta) \equiv 1 - \beta(\eta) = \frac{\kappa(1 - \sigma)H(\phi^*)}{1 - (1 - \kappa)(1 - \sigma)H(\phi^*)}, \quad (7)$$

where $C(\sigma, \eta)$ is the fraction of type- η industries cartelized. Finally, the derivation of the entire population of industries is performed by integrating the type- η distribution over $\eta \in [\underline{\eta}, \bar{\eta}]$. The mass of cartelized industries, which we refer to as the cartel rate $C(\sigma)$, is then defined by:

$$C(\sigma) \equiv \int_{\underline{\eta}}^{\bar{\eta}} [1 - \beta(\eta)] g(\eta) d\eta = \int_{\underline{\eta}}^{\bar{\eta}} \left[\frac{\kappa(1 - \sigma) H(\phi^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi^*(\sigma, \eta))} \right] g(\eta) d\eta. \quad (8)$$

3.3 Equilibrium Probability of Paying Penalties

Recall that $\sigma = qrs$ where q is the probability of a cartel being discovered, r is the probability that the AA investigates a reported case, and s is the probability of it succeeding with the investigation. We now want to derive the equilibrium value of s , where $s = p(\lambda L + R)$, L is the mass of leniency cases, and R is the mass of non-leniency cases handled by the AA. As both L and R depend on the cartel rate C and the cartel rate depends on s (through σ), this is a fixed point problem. We need to find a value for s , call it s' , such that, given $\sigma = qrs'$ (and recall that q is fixed and, for the time being, r is fixed), the induced cartel rate $C(qrs')$ is such that it generates L and R so that $p(\lambda L + R) = s'$.

With our expression for the cartel rate, we can provide expressions for L and R . The mass of cartel cases generated by the leniency program is:

$$L(\sigma) = \begin{cases} 0 & \text{if } qrs \leq \theta \\ \int_{\underline{\eta}}^{\bar{\eta}} (1 - H(\phi^*(qrs, \eta))) C(qrs, \eta) g(\eta) d\eta & \text{if } \theta < qrs \end{cases} \quad (9)$$

In (9), note that an industry does not apply for leniency when it is still effectively colluding. When collusion stops, leniency is used when the only equilibrium is that all firms apply for leniency, which is the case when $\theta < qrs$. Thus, when $\theta < qrs$, L equals the mass of cartels that collapse due to a high realization of π . This is consistent with a concern expressed by a European Commission official that many leniency applicants are from dying cartels.^{14, 15}

¹⁴This statement was made by Olivier Guersent at the 11th Annual EU Competition Law and Policy Workshop: Enforcement of Prohibition of Cartels in Florence, Italy in June 2006.

¹⁵That either all firms or no firms apply for leniency is a property of not only our analysis but all previous analyses on leniency programs (see the research reviewed in Spagnolo, 2008). What is lacking in this body of work is private information between cartel members which could potentially explain why one firm, but not others, would come forward to the AA. Of further note is that almost all other analyses do not have leniency being used in equilibrium. In a fixed environment, if collusion is initially stable then it is always stable in which case it never breaks down and leniency is never used. By allowing the environment to change with respect to market conditions, our model produces leniency applications in equilibrium. This property also arises in Harrington (2008) where it is assumed that σ is stochastic from period to period (though, contrary to the current model, σ is exogenous).

The mass of cartel cases generated without use of the leniency program is

$$R(\sigma) = \begin{cases} qrC(qrs) & \text{if } qrs \leq \theta \\ qr \int_{\underline{\eta}}^{\bar{\eta}} H(\phi^*(qrs, \eta)) C(qrs, \eta) g(\eta) d\eta & \text{if } \theta < qrs \end{cases} \quad (10)$$

If the leniency program is not being used (that is, $qrs \leq \theta$), then the mass of cases being handled by the AA is $qrC(qrs)$. If instead $\theta < qrs$, so that dying cartels do use the leniency program, then the cartels left to be caught are those which have not collapsed in the current period which is $\int_{\underline{\eta}}^{\bar{\eta}} H(\phi^*(qrs, \eta)) C(qrs, \eta) g(\eta) d\eta$. This expression comes from the fact that a fraction $C(qrs, \eta)$ of type- η industries are cartelized and a fraction $H(\phi^*(qrs, \eta))$ of them get a realization of π that allows them to continue to collude. Multiplying by qr , we have the mass of non-leniency cases.

The fixed point for the probability of AA success is defined by a value for s satisfying:

$$s = \begin{cases} p(qrC(qrs)) & \text{if } qrs \leq \theta \\ p\left(\lambda \int_{\underline{\eta}}^{\bar{\eta}} (1 - H(\phi^*(qrs, \eta))) C(qrs, \eta) g(\eta) d\eta\right) \\ \quad + qr \int_{\underline{\eta}}^{\bar{\eta}} H(\phi^*(qrs, \eta)) C(qrs, \eta) g(\eta) d\eta & \text{if } \theta < qrs \end{cases} \quad (11)$$

where we have substituted for L using (9) and R using (10). A fixed point is denoted $s^*(r)$ where we explicitly recognize its dependence on r since it will be solved for next.

3.4 Optimal Antitrust Policy

Ideally, an AA would choose its caseload - which is controlled by r (the fraction of non-leniency cases pursued) - so as to minimize the cartel rate in the economy: $\min_{r \in [0,1]} C(qrs^*(r))$. However, such an objective is problematic. Unless those working for the AA are of a benevolent species, a career concerns perspective would tell us that the AA will not seek to minimize the cartel rate because it is not observable, much less verifiable. As AA employees cannot be rewarded based on something that is unobservable, presumably their behavior is not driven by the cartel rate.

We will assume that AA employees are rewarded according to observable performance measures. One such measure is the fraction of cases won. The problem with that measure is the AA will then prosecute a very small number of cases and pour lots of resources into them in order to produce a high conviction rate. (Or, in a richer model, the AA may only take on the really easy cases.) A more reasonable measure is the mass of successful cases. Since leniency cases are presumed to be won for sure and a fraction s of non-leniency cases are won then $L + sR$ measures the mass of successful cases. The AA will then be modelled as choosing $r \in [0, 1]$ to maximize

$L(qrs^*(r)) + s^*(r)R(qrs^*(r))$, which takes the form:

$$\left\{ \begin{array}{ll} qrp(qrC(qrs^*(r)))C(qrs^*(r)) & \text{if } qrs^*(r) \leq \theta \\ \int_{\underline{\eta}}^{\bar{\eta}} (1 - H(\phi^*(qrs^*(r), \eta))) C(qrs^*(r), \eta) g(\eta) d\eta & \text{if } \theta < qrs^*(r) \\ +qr \int_{\underline{\eta}}^{\bar{\eta}} H(\phi^*(qrs^*(r), \eta)) C(qrs^*(r), \eta) g(\eta) d\eta \times & \\ p \left(\lambda \int_{\underline{\eta}}^{\bar{\eta}} (1 - H(\phi^*(qrs^*(r), \eta))) C(qrs^*(r), \eta) g(\eta) d\eta \right. & \\ \left. +qr \int_{\underline{\eta}}^{\bar{\eta}} H(\phi^*(qrs^*(r), \eta)) C(qrs^*(r), \eta) g(\eta) d\eta \right) & \end{array} \right. \quad (12)$$

When $qrs^*(r) \leq \theta$, the leniency program is not used so the total mass of cases is made up of those cartels prosecuted without leniency, $R = qrC(qrs^*(r))$, of which a fraction $p(R)$ are won. When leniency is used, which occurs when $\theta < qrs^*(r)$, the probability of success with non-lenieny cases is $p(\lambda L + R)$.

4 Results

Equilibrium was solved for a baseline parameter configuration and variations off of that baseline; the range of parameter values are provided in Table 1.¹⁶ Table 2 reports the results for the baseline configuration, both when there is no leniency program (Table 2a) and when full leniency is available (Table 2b). For enforcement policies $r \in \{0, .1, \dots, 1\}$, the reported variables are the equilibrium values for the probability of conviction, $s^*(r)$; the probability of paying penalties, $q \times r \times s^*(r)$; the mass of leniency cases, $L(qrs^*(r))$; the mass of non-lenieny cases, $R(qrs^*(r))$; the performance measure of the AA, $AA \equiv L(qrs^*(r)) + s^*(r) \cdot R(qrs^*(r))$; the cartel rate or fraction of industries cartelized, $C(qrs^*(r))$; the probability that a cartel is penalized, $\frac{L(qrs^*(r)) + s^*(r) \cdot R(qrs^*(r))}{C(qrs^*(r))}$; and the average duration of a cartel.

The case of no antitrust enforcement is $r = 0$ so that the AA does not prosecute any of the non-lenieny cases. It also means that no firm applies for leniency because there is no concern of being convicted otherwise. Thus, there is no enforcement when $r = 0$, with or without a leniency program. In this pre-Sherman Act pre-Article 81 world, almost 33% of industries are cartelized at anytime; the remainder do not collude because it is not incentive compatible or they have not had the opportunity to form a cartel since their most recent one collapsed.¹⁷ The average duration of a cartel is 156 periods with a cartel's demise solely being due to internal collapse as a result of strong market conditions.

Now suppose the AA has a mild enforcement policy in that it prosecutes 10% of reported cartels, $r = .1$. When there is no leniency program, 23% of industries are cartelized; hence, enforcement has reduced the cartel rate from its laissez faire level of 33%. 20% of cartels are discovered in any period (as $q = .2$), 10% are prosecuted

¹⁶The numerical methods used are described in the Appendix.

¹⁷In a richer model, we may want to allow the stochastic process on the opportunity to form a cartel to depend on the enforcement regime. Managers may be more brazen and less inhibited about communicating if enforcement is weaker. This would entail endogenizing κ .

(as $r = .1$), and the AA succeeds in 69% of those cases (see s^* in Table 2a). The compounding of these three probabilities means that there is a 1.4% chance each period that a cartel will be caught and convicted or, in short, penalized. The average cartel duration has dropped from 156 periods (when there is no enforcement) to 43 periods.

To identify the AA's optimal enforcement policy, we need to find the value for r that maximizes AA . The maximal fraction of industries that are successfully prosecuted is .00556 and is achieved when the AA investigates 60% of reported cartels. With that policy, about 9% of industries are cartelized at anytime and each of them faces a 6% chance of being penalized in any period. Average cartel duration is only 12 periods.

Table 2b reports results for when there is a leniency program in which the first firm to come forward has all penalties waived ($\theta = 0$). With a leniency program, the optimal policy of the AA is to prosecute 30% of the non-lenieny cases and this results in an 8% cartel rate. In any period, a cartel faces a 5.6% chance of being penalized. This penalization rate comes from the leniency program and also conviction through other means. Average cartel duration is 19 periods.

The ensuing analysis will work through three steps. First, the effect of a leniency program on the cartel rate is examined holding fixed the enforcement policy, r . Recall that the enforcement policy refers to the fraction of cartels reported outside of the leniency program which the AA chooses to prosecute. Second, how the AA adjusts its optimal enforcement policy in response to having a leniency program is then characterized. Third, the effect of a leniency program on the cartel rate is investigated while allowing the AA to appropriately adjust its enforcement policy. The key parameters that we vary are ρ (a higher value for which extends the range of caseloads for which the probability of conviction is concave in the caseload), v (which impacts the sensitivity of the probability of a conviction to the caseload), λ (which measures the burden of prosecuting a leniency case relative to a non-lenieny case), and α (which measures the profitability of the non-collusive solution relative to the collusive solution). Preliminary numerical runs revealed that ρ is a particularly influential parameter so we vary v , λ , and α for all $\rho \in \{1.3, 1.5, 1.7\}$.

Figure 1 provides information relevant to assessing the impact of a leniency program when the enforcement policy is unchanged. For policies ranging from no enforcement to prosecuting all reported cartels, the cartel rate is reported for the baseline parameterization ($\rho = 1.5$) and also $\rho = 1.3, 1.7$. The cases of no leniency ($\theta = 1$), partial leniency ($\theta = .05, .1$), and full leniency ($\theta = 0$) are considered.¹⁸ For any enforcement policy, the introduction of a full leniency program always reduces the cartel rate. A partial leniency program either has no effect - as it is not utilized by cartels - or it reduces the cartel rate. That a leniency program reduces the cartel rate is confirmed for other parameter values in Figures 2-4 (where we only compare full and no leniency). Unambiguously, we find that a leniency program is effective against collusion.

¹⁸Note that leniency is ineffective when $\theta > qrs^*(r)$ as then the only equilibrium is for firms not to apply for leniency. Thus, we only considered low values for θ since $qrs^*(r)$ is generally low.

Property 1 Given the antitrust authority’s enforcement policy (that is, r is fixed), the introduction of a leniency program reduces the frequency of cartels.

To examine Property 1 in greater depth, let us draw upon Table 2 which reports results for the baseline configuration. Going hand in hand with the cartel rate declining is that the probability a cartel assigns to being penalized (either through the leniency program or other means) goes up when a leniency program is put in place. For example, when $r = .5$, the introduction of a leniency program increases the probability of a cartel paying penalties from .051 to .095. When there is no leniency program, the probability of paying penalties is the compounding of the probability of being discovered (which is $q = .2$), of being prosecuted (which is r), and of being convicted, which is endogenous and depends on the caseload. When there is a leniency program, the probability of paying penalties is the sum of the previous probability (though it’ll take a different value because the caseload is different) and the probability of applying for leniency, which is the probability that the cartel internally collapses (as all dying cartels apply for leniency). Thus, unless the probability of succeeding in a non-lenieny case is sufficiently lower when there is a leniency program, the probability of a cartel being penalized will be higher because now a cartel can be penalized by being caught and convicted *or* by applying for leniency after the cartel has collapsed. In fact, with a leniency program in place, the AA not only shuts down cartels by virtue of the leniency program, but the AA is also more successful in winning non-lenieny cases. For example, when $r = .5$, the probability of winning a non-lenieny case, s^* , rises from .512 to .711. This higher success rate is most likely due to a combination of fewer cartels - and thus fewer cases to prosecute - and that some cases are handled through the leniency program which uses up fewer resources, thereby leaving more resources to try non-lenieny cases.

Holding the enforcement policy fixed, a leniency program always increases the chances of a cartel being penalized, but it may or may not reduce the average cartel duration. In Table 2, if $r = .5$ then the introduction of a leniency program causes the average cartel duration to fall from 14.2 to 10.8 periods. When instead enforcement policy is weak ($r = .1$), the average cartel duration rises from 42.5 to 46.3 periods. With the latter, note that the cartel rate still declines, from 23% to 16.3%. We will later return to the issue of cartel duration.

To explore the impact of a leniency program on the aggressiveness of enforcement by the AA, Figure 5 reports the optimal enforcement policy for various parameter configurations. Let us begin with Figure 5a. With the exception of $\rho = 1.2$, the AA takes on a smaller fraction of non-lenieny cases after a leniency program is instituted.¹⁹ For example, for $\rho = 1.5$, it takes on 60% of such cases when there is no leniency program, but only 30% when there is a leniency program. Given that the AA is required to take on all leniency cases, those cases use up resources which, through the probability of success function $p(\lambda L + R)$, lowers the probability of achieving success in a non-lenieny case. However, at the same time, there are fewer cartels when there is a leniency program, holding r fixed (Property 1), which could allow the

¹⁹There is also the exception of $\rho = 1.1$ in that there are multiple optima when there is no leniency program.

AA to take on a larger fraction of non-leniency cases. Indeed, this is what occurs for $\rho = 1.2$, but that case is more of an exception. Allowing for different values of v , λ , and α from the baseline configuration, we again find that a leniency program results in the AA prosecuting a smaller fraction of non-leniency cases.

Property 2 Generally, the introduction of a leniency program results in the antitrust authority pursuing a less aggressive enforcement policy in that it prosecutes a smaller fraction of cartels discovered outside of the leniency program.

Next we turn to the issue of greatest interest: Once the AA adjusts its enforcement policy, what is the impact of a corporate leniency program on the frequency of cartels? Let us begin by examining Table 3. First consider $\rho = 1.3$. In the pre-leniency environment, the AA prosecutes 60% of its cases. The resulting cartel rate is 20.3% and the per period chances that a cartel is penalized is 1.9%. Introducing a leniency program causes the AA to lower its investigation rate so that it pursues 40% of non-leniency cases. Still, the penalization rate is higher at 4.5% and cartel duration is shorter at 24 periods as compared to 33 periods. Most importantly, the cartel rate is halved from 20.3% to 10.1% with the leniency program. Now suppose $\rho = 1.5$. Introducing a leniency program substantially reduces the fraction of non-leniency cases prosecuted from 60% to 30%. The penalization rate falls from 6.1% to 5.6%, and cartel duration rises from 12 to 19 periods. Nevertheless, a leniency program lowers the cartel rate from 9.1% to 8.1%. Finally, suppose $\rho = 1.7$. In the absence of a leniency program, the AA prosecutes 40% of reported cartels, while enforcement declines to 20% with a leniency program. The weaker enforcement lowers the penalization rate from 5.7% to 4.4%, and cartel duration rises significantly from 13 to 24 periods. The most striking finding is that there are now more cartels in the economy. Introducing a leniency program *raises* the cartel rate from 9.9% to 10.5%.

That a leniency program can result in more cartels is not a pathological finding for our model. Figure 6 compares the cartel rate with and without leniency for a wide range of parameter values. Though it is difficult to identify any systematic relationship between these parameters and the incremental impact of a leniency program on the cartel rate, it is observed that a leniency program often ends up raising the cartel rate. Additional results are reported in Tables 4-9 where we find that whenever the cartel rate rises in response to a leniency program, there is also an increase in average cartel duration.

Property 3 When the antitrust authority chooses its optimal enforcement policy, the introduction of a leniency program can either lower or raise the cartel rate, depending on the parameter configuration. It can raise the cartel rate because the antitrust authority less aggressively pursues those cases not generated through the leniency program. Whenever the introduction of a leniency program raises the cartel rate, average cartel duration also rises.

To better understand how a leniency program is raising the cartel rate, it is crucial to take account of industry heterogeneity. Recall that industries differ in the

parameter η which controls the short-run profit gain from a cartel member deviating from the collusive outcome. Thus, when η is higher, a cartel is less stable in the sense that it'll internally collapse for a wider set of market conditions, which implies shorter average duration. In fact, when η is sufficiently high, a cartel never forms in that it is not incentive compatible for any market conditions. $\hat{\eta}$ denotes the highest value for η such that a cartel forms with positive probability.

Figure 7 reports the average cartel duration for each industry type $\eta \leq \hat{\eta}$. An examination reveals two notable effects from introducing a leniency program. First, it reduces $\hat{\eta}$ and thereby shrinks the range of industry types for which a cartel forms with positive probability; in Figure 7b, $\hat{\eta}$ falls from about 1.5 to 1.35. This finding holds for all of our parameterizations, as evidenced by the output in Tables 3-9; $\hat{\eta}$ is always lower with a leniency program. Second, for those industries that do cartelize with positive probability (that is, $\eta \leq \hat{\eta}$), the average duration of a cartel could be shorter ($\rho = 1.3$) or longer ($\rho = 1.7$). Other parameterizations - see Figures 8-10 - confirm that finding. In sum, the institution of a leniency program causes fewer cartels to form but those that do form could last for a longer time.

To understand the determination of duration, recall that a cartel shuts down when it internally collapses (due to strong market conditions) or it is discovered and successfully prosecuted by the AA; the latter occurs with probability $\sigma^*(\hat{r}) \equiv q\hat{r}s^*(\hat{r})$ where \hat{r} is the AA's optimal enforcement policy. Holding r fixed, a leniency program can make collusion less profitable because, in response to a cartel collapsing, all firms apply for leniency which can raise expected penalties.²⁰ This reduces collusive value which means market conditions don't have to be as strong for collusion to no longer be incentive compatible. With internal collapse being more likely, average cartel duration is shorter. This effect, however, is weaker for industries with a lower value for η . Since cartels in those industries are more stable, they are less likely to collapse and thus the leniency program is less likely to be used. All this adds up to a leniency program not raising expected penalties as much for industries with a lower value for η . In unreported results, we show that, holding the enforcement policy fixed, the availability of leniency decreases average cartel duration for all industry types, but the decrease isn't as large when η is lower.

Thus far, we've argued that a leniency program reduces cartel duration, holding the enforcement policy fixed. To explain how a leniency program can actually increase cartel duration - as shown, for example, in Figure 7b - we need to take account of how the AA adjusts its enforcement policy in response to having a leniency program. By Property 2, we know that the AA will lower r which can mean weaker enforcement in terms of a lower probability a cartel is successfully prosecuted by the AA without use of leniency. That effect is clearly beneficial to cartels from all industries but is especially advantageous for industries with low values of η because a more stable cartel is more concerned with detection via non-leniency means than through the leniency program. For consider the extreme case when η is so low that a cartel in

²⁰In response to the discontinuation of collusion, firms want to minimize expected penalties. It is not an equilibrium for all firms not to apply since, with full leniency, any single firm can avoid all penalties by applying. But once one firm is expected to apply then it is optimal for all to do so. This is the Race to the Courthouse effect.

such an industry never collapses on its own, so shutdown occurs only when it is caught and convicted by the AA. The leniency program's only effect is then through a weaker enforcement policy, in which case collusion is easier for those cartels and this translates into longer duration. Consistent with this explanation, the increase in cartel duration from a leniency program is greater when η is lower and thus cartels are more stable (see, for example, Figure 7c).

Property 4 When the antitrust authority chooses its optimal enforcement policy, the introduction of a leniency program: 1) reduces the range of industries that cartelize; and 2) can either raise or lower the average cartel duration, depending on the parameter configuration and the industry type. A leniency program reduces average cartel duration or eliminates cartels altogether in industries with a high value of η so that cartels are relatively unstable. A leniency program can increase average cartel duration in industries with a low value of η so that cartels are relatively stable.

Key to a leniency program raising the cartel rate is that the AA chooses to investigate fewer cases generated outside of the leniency program. The basis for this behavioral response comes from the performance measure used by the AA. While we assume it is the number of successful cartel cases, we suspect similar findings would arise if the AA focused on the amount of fines collected. What this type of objective implies is that the AA would not want to have a policy so effective that it served to largely eliminate cartels as it would then mean very few successes for the AA. In response to having a leniency program, the AA then reduces non-lenieny enforcement both because it can raise the success rate - which contributes to winning more cases - and because it can weaken deterrence, promotes cartels, and thereby enhances the number of successful cases.

It is natural to ask how realistic is this finding. Our view is that while its empirical validity is an open question, the finding is the implication of a plausible assumption about the objective of the AA. Furthermore, this objective is consistent with the public speeches and testimony of officials of the Antitrust Division of the U. S. Department of Justice, which emphasize the total number of successful investigations, the amount of fines collected, and the length of prison sentences. This perspective is clear from testimony before Congress by the head of the Antitrust Division.

Joel I. Klein, Assistant Attorney General (May 4, 1999): "As a result of our aggressive overall criminal enforcement efforts against hard-core antitrust violations such as price-fixing and market allocation, we have set records in the last two fiscal years in the level of fines collected. In fiscal year 1997, criminal fines totaling \$205 million dollars were secured in cases brought by the Antitrust Division. This total is five times higher than during any previous year in the Division's history."²¹

²¹Statement before the Subcommittee on Antitrust, Business Rights, and Competition, Committee on the Judiciary, United States Senate concerning International Antitrust Enforcement, May 4, 1999.

Thomas O. Barnett, Assistant Attorney General (September 25, 2007): "The Division's cartel enforcement efforts were outstanding for Fiscal Year 2007. The Division set a record for the most total jail time imposed (almost 30,000 jail days); obtained the second highest amount of fines in the Division's history (over \$630 million); and succeeded in obtaining the longest jail sentence for a foreign national charged with an antitrust offense (14 months). These accomplishments reflect great strides in the Division's efforts to rid the marketplace of cartels and their harm to consumers."²²

Note that Assistant Attorney General Joel Klein highlighted the record high fines collected and still, ten years later, Assistant Attorney General Thomas Barnett praised the continued increase in fines and jail time. There is no recognition that the rising penalties is not only consistent with more aggressive enforcement but also with there being more cartels fueling more cases. Indeed, successful enforcement (and a successful leniency program) ought to eventually imply fewer cases. Yet, just two years ago, another DOJ official was noting the record number of grand jury investigations:

Scott D. Hammond, Deputy Assistant Attorney General for Criminal Enforcement (March 26, 2008): "At the close of Fiscal Year 2007, the Division had 135 pending grand jury investigations, including over 50 investigations of suspected international cartel activity. This is the highest number of pending grand jury investigations since 1992."²³

Almost 15 years after the revision of the Corporate Leniency Program, grand jury investigations continue to increase. All this suggests that the emphasis of the DOJ is on the aggregate amount of anti-cartel activity as measured by the number of cases and the amount of penalties, which supports the key assumption about preferences which drives our main result.

In concluding, the introduction of a leniency program is found to be highly detrimental to marginally stable cartels as they no longer form. Firms in those industries are deterred from colluding by the prospect of cartel members using leniency in the future when the cartel collapses. This effect serves to reduce the cartel rate. At the same time, a leniency program can result in highly stable cartels having even longer lifetimes because the AA chooses to pursue less aggressively those cases produced outside of the leniency program. It is the longer lifetime of those cartels that results in a leniency program producing a higher cartel rate. Fewer cartels form but those that do form last for a longer time, which translates, at the steady-state, into a higher frequency of cartels at any moment in time.

²²Statement before the Task Force on Antitrust and Competition, Committee on the Judiciary, United States House of Representatives, concerning Oversight of the United States Department of Justice, Antitrust Division, September 25, 2007.

²³"Recent Developments, Trends, and Milestones In The Antitrust Division's Criminal Enforcement Program." Speech delivered at the 56th Annual Spring Meeting Presented by the ABA Section of Antitrust Law, March 26, 2008.

5 Concluding Remarks

There are two primary contributions of this paper. First, it provides a more comprehensive assessment of the effects of a corporate leniency program on shutting down cartels and deterring them from forming. After the introduction of a leniency program, we find that the antitrust authority is less aggressive in its enforcement policy with respect to the investigation of cases not involving leniency, and this effect can be so severe as to cause the frequency of cartels to be higher. A surfeit of leniency applications need not imply fewer cartels.

A second and more general contribution is to offer a theoretical testbed for exploring the impact of anti-cartel policies; both those currently in use and under consideration. This framework is unique in: 1) encompassing a stochastic process describing cartel formation and dissolution; 2) taking account of the general equilibrium effects of a policy through its influence on the antitrust authority's caseload via the number of cartel cases; and 3) modelling how a self-interested antitrust authority will adjust other dimensions of enforcement to a new policy. To convey the broad applicability of this framework, let us suggest some additional policy issues that can be investigated using it.

- In response to introducing a leniency program, should the budget of the antitrust authority be expanded? The question is whether the marginal decline in the cartel rate from a bigger budget is higher or lower after a leniency program is adopted. This issue can be explored by varying v which controls the relationship between caseload and the probability of success, and thus implicitly captures the antitrust authority's resource constraint.
- After a flood of leniency applications with its 2002 revision, the European Commission adopted new procedures in June 2008 which will allow these applications to be handled in a more expedited manner and involving fewer resources. The impact of such procedures can be investigated by lowering λ which measures the ratio of resources used for a leniency case relative to a non-lenieny case.
- An enforcement dimension that has received some discussion in recent years is screening industries for suspected cartels; see, for example, Harrington (2007) and Friederiszick and Maier-Rigaud (2008). The impact of such a program - including how it interacts with a leniency program - can be examined by changing q which is the fraction of cartels discovered without a member having entered the leniency program.
- There is considerable variation across countries in terms of specific features of leniency programs. For example, the European Commission provides partial leniency for later applicants. In Japan, there is a significant decline in leniency if a firm comes forward after an investigation has started (only 30% of fines are waived) and, furthermore, leniency is the same whether the firm is the first or the third to apply. Within the framework developed here, the efficacy of different program features can be investigated with the goal of improving the design of leniency programs.

Finally, a critical avenue for future research is to develop a richer and better-grounded model of the antitrust authority and to investigate the extent to which its behavior coincides or diverges from what is in society's best interests. This is a major open question, both theoretically and empirically.²⁴

6 Appendix: Numerical Methods

We start by specifying the values for the 13 parameters – n , α , ω , θ , κ , δ , γ , τ , ξ , v , λ , q , and ρ – as well as the two probability distributions, $H(\pi)$ and $G(\eta)$, for market shocks and industry types, respectively. A log-normal distribution $LN(\mu, \sigma^2)$ with $\mu = 0$ and $\sigma = 1.5$ is used for both $H(\pi)$ and $G(\eta)$, though $H(\pi)$ is defined over $[1, \infty)$, while $G(\eta)$ is defined over $[1.1, \infty)$. Table 1 reports the range of feasible values for the parameters as well as the baseline set of values.

The numerical problem has a nested structure. Given a value of r , the underlying problem is to find a fixed point, $s^*(r)$, to $s = p(\lambda L(qrs) + R(qrs))$, where $L(qrs)$ is the mass of cartel cases generated by the leniency program defined in (9) and $R(qrs)$ is the mass of non-lenieny cartel cases defined in (10). Note that the dependence of L and R on r and s is through the endogenous probability of paying penalties, $\sigma = qrs$, which affects the incentive compatibility of collusion.

The procedure for finding $s^*(r)$ begins by specifying an initial value for s . For each η , we need to solve for a fixed point to $\psi(Y, qrs, \eta)$,

$$Y^*(qrs, \eta) = \psi(Y^*(qrs, \eta), qrs, \eta).$$

As there may be multiple fixed points, we use the Pareto criterion to select among them and thus choose the largest fixed point. Since $\psi(Y, qrs, \eta)$ is increasing and $\psi(\mu, qrs, \eta) < \mu$ then, by setting $Y^0 = \mu$ and iterating on $Y^{t+1} = \psi(Y^t, qrs, \eta)$, this process converges to the largest fixed point, $Y^*(qrs, \eta)$.

In computing the stationary distributions from Section 3.2, we need to take the step of computationally searching for $\hat{\eta}(qrs)$ which is the smallest industry type for which collusion is not incentive compatible for any market condition. $\hat{\eta}(qrs)$ is defined by: $Y^*(qrs, \eta) > \alpha\mu$ for $\underline{\eta} < \eta \leq \hat{\eta}(qrs)$ and $Y^*(qrs, \eta) = \alpha\mu$ for $\eta > \hat{\eta}(qrs)$. To perform this step, we set $\underline{\eta} = 1.1$ and $\bar{\eta} = 10$ and use a 1000 element finite grid of values for η , denoted $\bar{\Gamma}(\underline{\eta}, \bar{\eta})$. $\hat{\eta}(qrs)$ is located by applying the iterative bisection method on $\bar{\Gamma}(\underline{\eta}, \bar{\eta})$. As part of the bisection method, $\bar{\eta}$ needs to be set at a sufficiently high value so that $Y^*(qrs, \bar{\eta}) = \alpha\mu$. Once having identified $\hat{\eta}(qrs)$ and using $Y^*(qrs, \eta)$ and (4), $\phi^*(qrs, \eta)$ is calculated for a finite grid over $[\underline{\eta}, \hat{\eta}(qrs)]$. These values are then used in computing $L(qrs)$ and $R(qrs)$. The integration uses the Newton-Cotes quadrature method with the trapezoid rule (see Miranda and Fackler, 2002).

²⁴In a very simple model, Harrington (2010) pursues this issue by characterizing the circumstances under which the behavior of an antitrust authority - which is interested in maximizing the number of successful cartel cases - coincides with that which minimizes the cartel rate.

Choosing an initial value for s and using our derived expressions for $L(qrs)$ and $R(qrs)$, we then compute:

$$\widehat{p}(qrs) \equiv p(L(qrs), R(qrs)) = \frac{\tau}{\xi + v [\lambda L(qrs) + R(qrs)]^\rho}. \quad (13)$$

After specifying a tolerance level ϵ , if $|s - \widehat{p}(qrs)| > \epsilon$ then a new value for s is selected using the iterative bisection method. Note that once a new value for s is specified, the entire preceding procedure must be repeated. This procedure is repeated until the process converges to the fixed point value of $s^*(r)$ such that

$$|s^*(r) - \widehat{p}(qrs^*(r))| \leq \epsilon. \quad (14)$$

ϵ is set at .001.²⁵

With the equilibrium probability of success, $s^*(r)$, we have the equilibrium probability of paying penalties, $qrs^*(r)$, from which we can calculate the stationary distribution on cartels using the formulas in Section 3.2, especially (8) which provides the frequency of cartels.

To derive the AA's optimal policy, we allow $r \in \{0, .1, \dots, 1\}$ and perform the procedure described above for each of those values. The AA's optimal policy is the value of r which maximizes $L(qrs^*(r)) + s^*(r)R(qrs^*(r))$. To test whether the coarseness of the grid was driving some results, we re-ran the model for a limited number of parameter configurations when $r \in \{0, .01, \dots, 1\}$. While quantitative results did change, none of the properties we describe below were altered.

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²⁵To explore whether the fixed point is unique, we plotted $\widehat{p}(qrs)$ for all $s \in \{0, .1, \dots, 1\}$ and $r \in \{0, .1, \dots, 1\}$; for $\rho \in \{1.3, 1.5, 1.7\}$ and $\theta \in \{0, 1\}$. In all cases, $\widehat{p}(qrs)$ is a concave function of s and there is no reason to think that there is more than one fixed point.

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Table 1: Parameters

Parameter	Feasible Range	Values Used in Simulations	Baseline Value	Description
α	$[0, 1)$	$\{0, .1, .2, \dots, .9\}$	0	Degree of competitiveness
q	$[0, 1]$	$\{.1, .2, .3\}$.2	Probability that a cartel is reported to the antitrust authority
n	$[1, \infty)$	4	4	Number of firms in an industry
θ	$[0, 1]$	$\{0, .05, .1, 1\}$	$\left\{ \begin{array}{l} 1 \text{ with no leniency} \\ .05 \text{ or } .1 \text{ with partial leniency} \\ 0 \text{ with full leniency} \end{array} \right.$	Leniency parameter
ω	$(\theta, 1)$	$\{.75, .7625, .775, 1\}$	$\left\{ \begin{array}{l} 1 \text{ with no leniency} \\ .7625 \text{ or } .775 \text{ with partial leniency} \\ .75 \text{ with full leniency} \end{array} \right.$	Leniency parameter ($= \frac{n-1+\theta}{n}$)
κ	$[0, 1]$	$\{.025, .05, .1\}$.05	Opportunity rate to cartelize
δ	$[0, 1)$	$\{.75, .85, .9\}$.85	Discount factor
γ	$[0, \infty)$	2	2	Damage multiple
$H(\pi)$	$[\underline{\pi}, \bar{\pi}] \rightarrow [0, 1]$ $0 < \underline{\pi} < \bar{\pi} \leq \infty$	Log-Normal Distribution ($\underline{\pi} = 1; \bar{\pi} = \infty$)	Log-Normal Distribution ($\underline{\pi} = 1; \bar{\pi} = \infty$)	Cdf on profit shocks
$G(\eta)$	$[\underline{\eta}, \bar{\eta}] \rightarrow [0, 1]$ $1 < \underline{\eta} < \bar{\eta} \leq \infty$	Log-Normal Distribution ($\underline{\eta} = 1.1; \bar{\eta} = \infty$)	Log-Normal Distribution ($\underline{\eta} = 1.1; \bar{\eta} = \infty$)	Cdf on industry types
τ	$(0, 1]$	$\{.6, .8, 1\}$.8	$p(L, R) \equiv \frac{\tau}{\xi+v(\lambda L+R)^\rho}$
ξ	$\geq \tau$	1	1	parameter in $p(L, R)$
v	$[0, \infty)$	$\{100, 200, \dots, 900, 1000\}$	500	parameter in $p(L, R)$
λ	$[0, 1]$	$\{.1, .2, \dots, .9, 1\}$.5	parameter in $p(L, R)$
ρ	$[1, \infty)$	$\{1.0, 1.1, \dots, 1.9, 2.0\}$	1.5	parameter in $p(L, R)$

Table 2: Baseline Results
 $(\rho = 1.5)$

(a) no leniency ($\theta = 1$)

r	$s^*(r)$	$q \times r \times s^*(r)$	$L(qrs^*(r))$	$R(qrs^*(r))$	AA	cartel rate	probability cartel is penalized	avg. duration of a cartel
0.	.800781	0.	0.	0.	0.	.326347	0.	155.568
.1	.691406	.0138281	0.	.0045959	.00317763	.229795	.0138281	42.5436
.2	.613281	.0245313	0.	.00721283	.0044235	.180321	.0245313	26.9193
.3	.5625	.03375	0.	.00896896	.00504504	.149483	.03375	20.4948
.4	.529297	.0423438	0.	.0101603	.00537783	.127004	.0423438	16.7763
.5	.511719	.0511719	0.	.0108177	.00553563	.108177	.0511719	14.1663
.6	.507813	.0609375	0.	.0109584	.00556482	.0913201	.0609375	12.097
.7	.519531	.0727344	0.	.0105471	.00547952	.0753361	.0727344	10.2748
.8	.546875	.0875	0.	.00951043	.00520102	.0594402	.0875	8.68384
.9	.578125	.104063	0.	.008388	.00484931	.0466	.104063	7.36874
1.	.615234	.123047	0.	.00712106	.00438112	.0356053	.123047	6.28207

(b) full leniency ($\theta = 0$)

r	$s^*(r)$	$q \times r \times s^*(r)$	$L(qrs^*(r))$	$R(qrs^*(r))$	AA	cartel rate	probability cartel is penalized	avg. duration of a cartel
0.	.800781	0.	0.	0.	0.	.326347	0.	155.568
.1	.707031	.0141406	.00172931	.00322326	.00400825	.162892	.0246068	46.2674
.2	.673828	.0269531	.00146561	.00447425	.00448049	.113322	.0395377	26.9705
.3	.666016	.0399609	.0013304	.00478344	.00451625	.0810545	.05557187	18.8118
.4	.681641	.0545313	.00114561	.0043661	.00412173	.0557219	.0739696	14.0095
.5	.710938	.0710938	.000915516	.00348462	.00339286	.0357617	.0948742	10.819
.6	.748047	.0897656	.000641522	.00236427	.0024101	.0203437	.118469	8.58553
.7	.775391	.108555	.000423745	.00141355	.0015198	.0105205	.14446	7.00619
.8	.789063	.12625	.000263309	.000736697	.000844609	.00486767	.173514	5.82197
.9	.798828	.143789	.0000908122	.000214449	.00026212	.0012822	.204431	4.91776
1.	.800781	.160156	.0000150377	.0000186852	.0000300005	.000108464	.276594	3.6154

Table 3: Impact of the Leniency Policy on Cartel Behavior
for $\rho \in \{1.0, 1.1, \dots, 1.9, 2.0\}$

ρ	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\hat{\eta}$	
	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
1.0	.9	.8	.280756	.198025	.005625	.0167513	80.2996	74.9596	1.7764	1.4916
1.1	1. (*)	.9	.265363	.173012	.0078125	.0218537	64.6766	52.9967	1.7586	1.4649
1.2	.5	.9	.240116	.133414	.0119141	.03268	47.6928	33.3012	1.7319	1.4204
1.3	.6	.4	.202972	.100516	.0192187	.0448551	32.8335	23.5487	1.6874	1.3759
1.4	.8	.3	.138895	.0938382	.0375	.0482288	18.701	21.8319	1.5895	1.367
1.5	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
1.6	.5	.2	.0870105	.106969	.0638672	.0420231	11.5838	25.2437	1.4916	1.3848
1.7	.4	.2	.0986173	.104659	.0565625	.0438959	12.9266	24.2417	1.5183	1.3848
1.8	.4	.2	.092889	.101227	.0596875	.0442401	12.358	23.8562	1.5005	1.3759
1.9	.4	.2	.0907345	.100516	.0614062	.0448551	12.0019	23.5487	1.5005	1.3759
2.0	.4	.2	.0886194	.10009	.0625	.0452272	11.8467	23.3676	1.4916	1.3759

* At $\rho = 1.1$, there are other optima at .4, .5, and .8 for the case of no leniency. However, they all yield the identical values for the variables reported above.

Table 4: Impact of the Leniency Policy on Cartel Behavior
for $v \in \{100, 200, \dots, 1000\}$

v	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\hat{\eta}$	
	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
100	.4	.2	.0922987	.101227	.0601562	.0442401	12.2589	23.8562	1.5005	1.3759
200	.4	.2	.0992805	.105277	.0560937	.0433882	13.0359	24.5019	1.5183	1.3848
300	.5	.2	.0863227	.107153	.0644531	.041878	11.4747	25.3169	1.4916	1.3848
400	.5	.2	.0956224	.111326	.0582031	.0410243	12.6165	26.0574	1.5094	1.3937
500	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
600	.7	.2	.0959345	.117878	.0579687	.0385085	12.6687	27.9204	1.5094	1.4026
700	.8	.3	.126689	.0879494	.0425	.051208	16.7166	20.4895	1.5717	1.3581
800	.6	.3	.158486	.0932563	.0309375	.0487899	22.0555	21.5996	1.6251	1.367
900	.5	.3	.176231	.0983806	.0257812	.0467452	25.7532	22.6667	1.6518	1.3759
1000	.4	.3	.190102	.101156	.0221875	.0443013	29.2032	23.8251	1.6696	1.3759

Table 7: Impact of the Leniency Policy on Cartel Behavior
for $\delta \in \{.75, .85, .9\}$

δ	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\hat{\eta}$	
	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.75	.5	.2	.0656841	.0770546	.0632812	.0496593	10.8533	22.0817	1.4293	1.3225
.85	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
.90	.6	.2	.119522	.131551	.0515625	.0357735	14.535	29.4743	1.6607	1.4293

Table 8: Impact of the Leniency Policy on Cartel Behavior
for $\kappa \in \{.025, .05, .1\}$

κ	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\hat{\eta}$	
	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.025	.4	.2	.073827	.0902084	.0521875	.0397653	14.4871	26.7658	1.5272	1.4471
.05	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
.1	1.	.2	.10126	.103686	.065625	.0440132	10.5324	24.5037	1.4827	1.3136

Table 9: Impact of the Leniency Policy on Cartel Behavior
for $\tau \in \{.6, .8, 1\}$

τ	Optimal Enforcement Policy		Cartel Rate		Penalization Rate		Cartel Duration		$\hat{\eta}$	
	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency	w/o leniency	w/ leniency
.6	.9	.3	.149483	.112574	.03375	.0400895	20.4948	26.6214	1.6162	1.3937
.8	.6	.3	.0913201	.0810545	.0609375	.0557187	12.097	18.8118	1.5005	1.3492
1	.4	.2	.0913201	.0923258	.0609375	.0496988	12.097	21.238	1.367	1.367

Figure 1 : Effects of Enforcement Policy on the Cartel Rate
(Baseline Case)

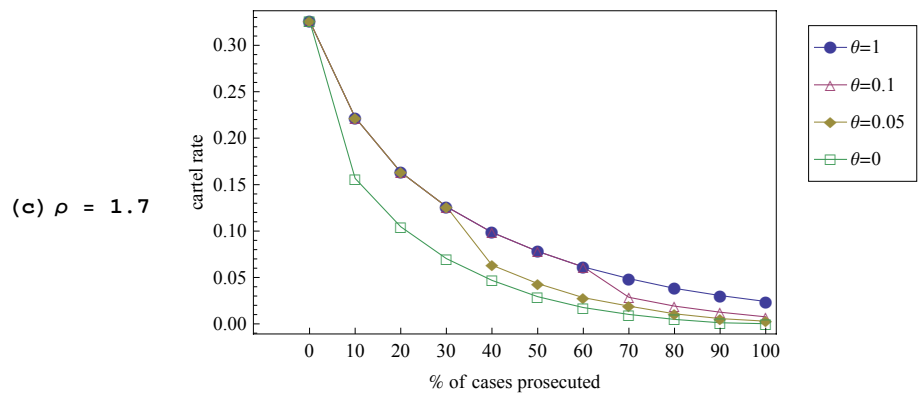
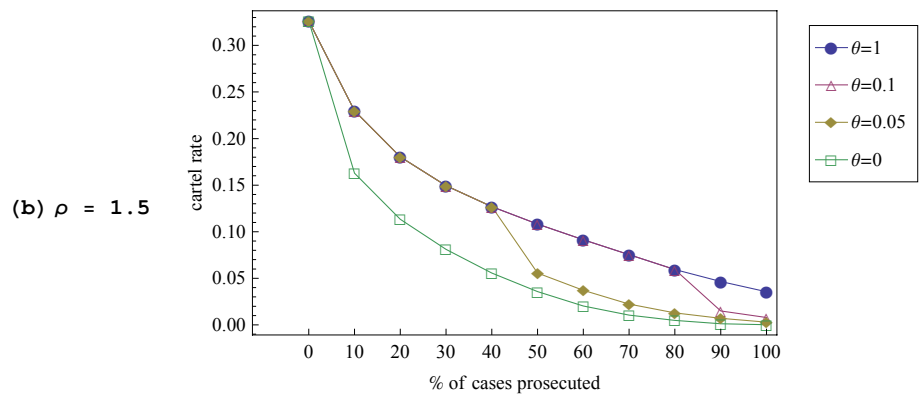
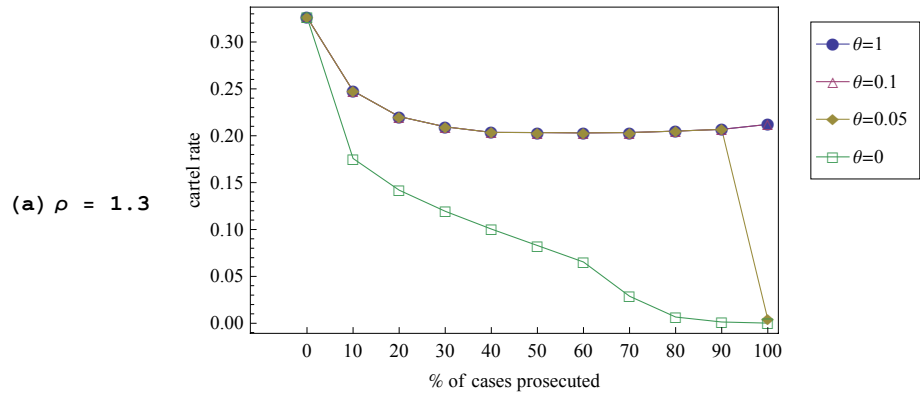


Figure 2 : Effects of Enforcement Policy on the Cartel Rate for $v \in \{100, 1000\}$

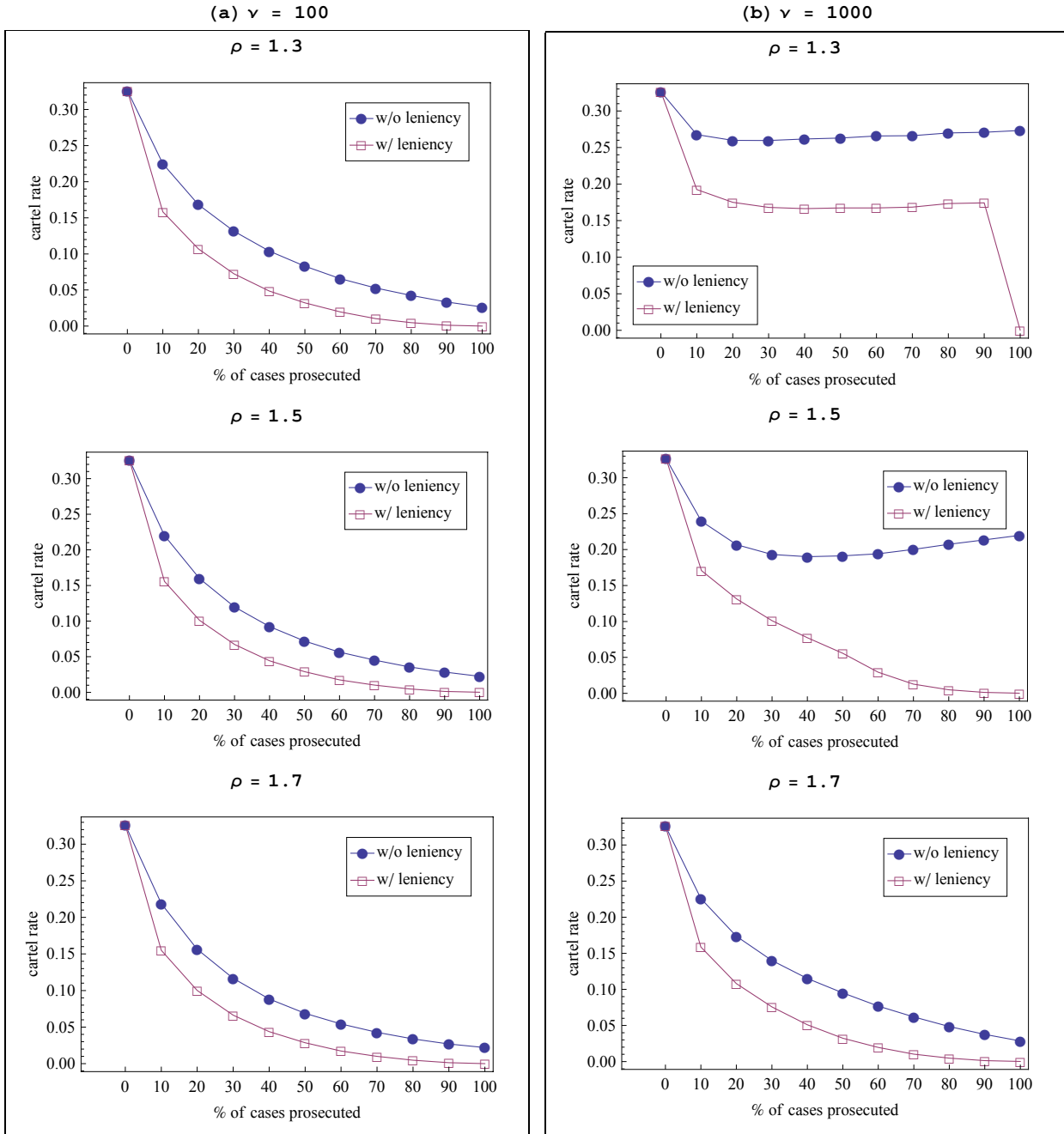


Figure 3 : Effects of Enforcement Policy on the Cartel Rate
for $\lambda \in \{0.2, 0.8\}$

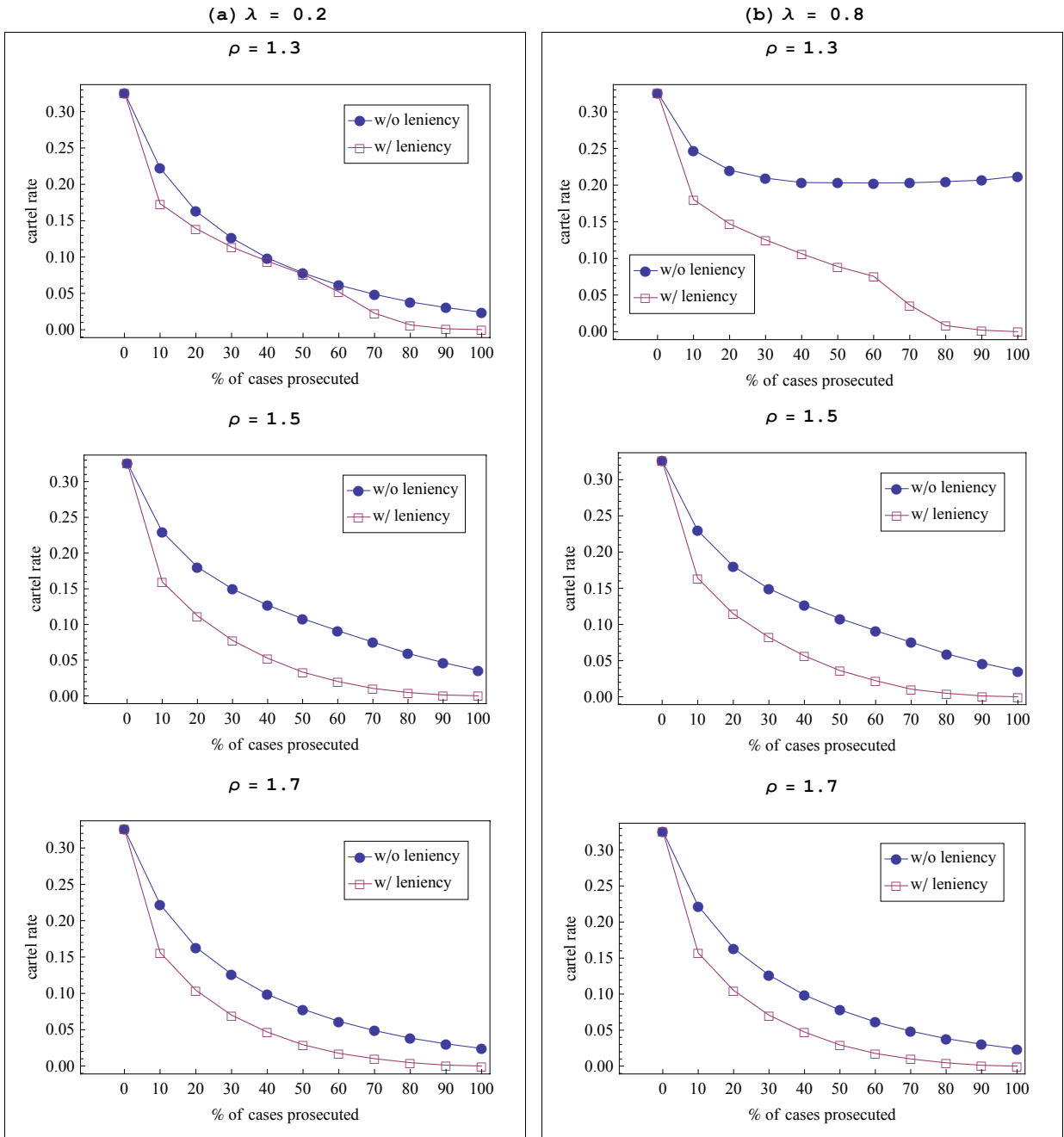


Figure 5 : Optimal Enforcement Policy with and without Leniency
for ρ , ν , λ , and α

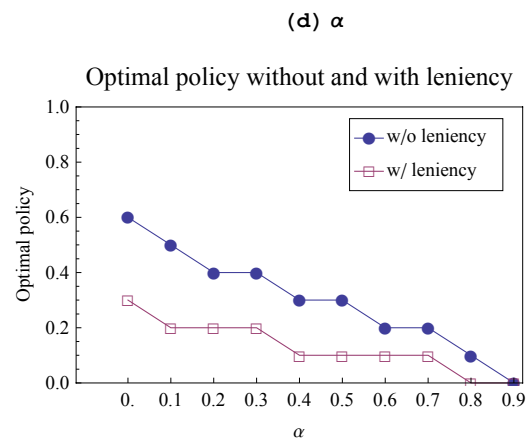
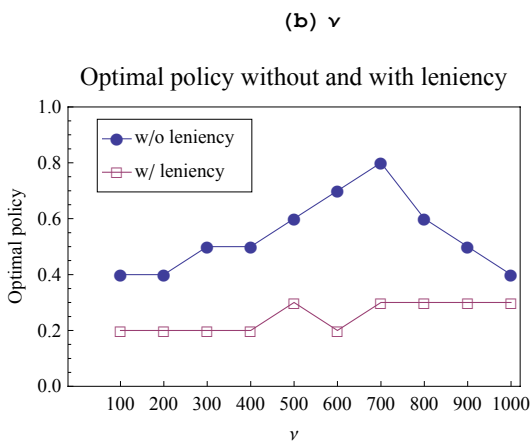
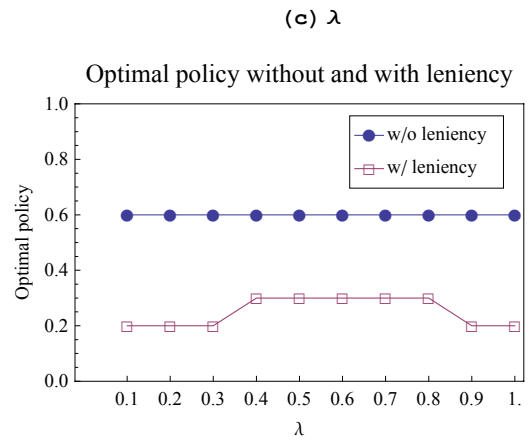
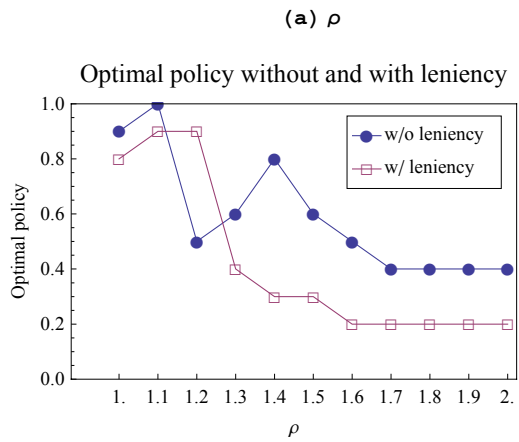


Figure 6 : Endogenous Cartel Rates with and without Leniency for ρ , ν , λ , and α

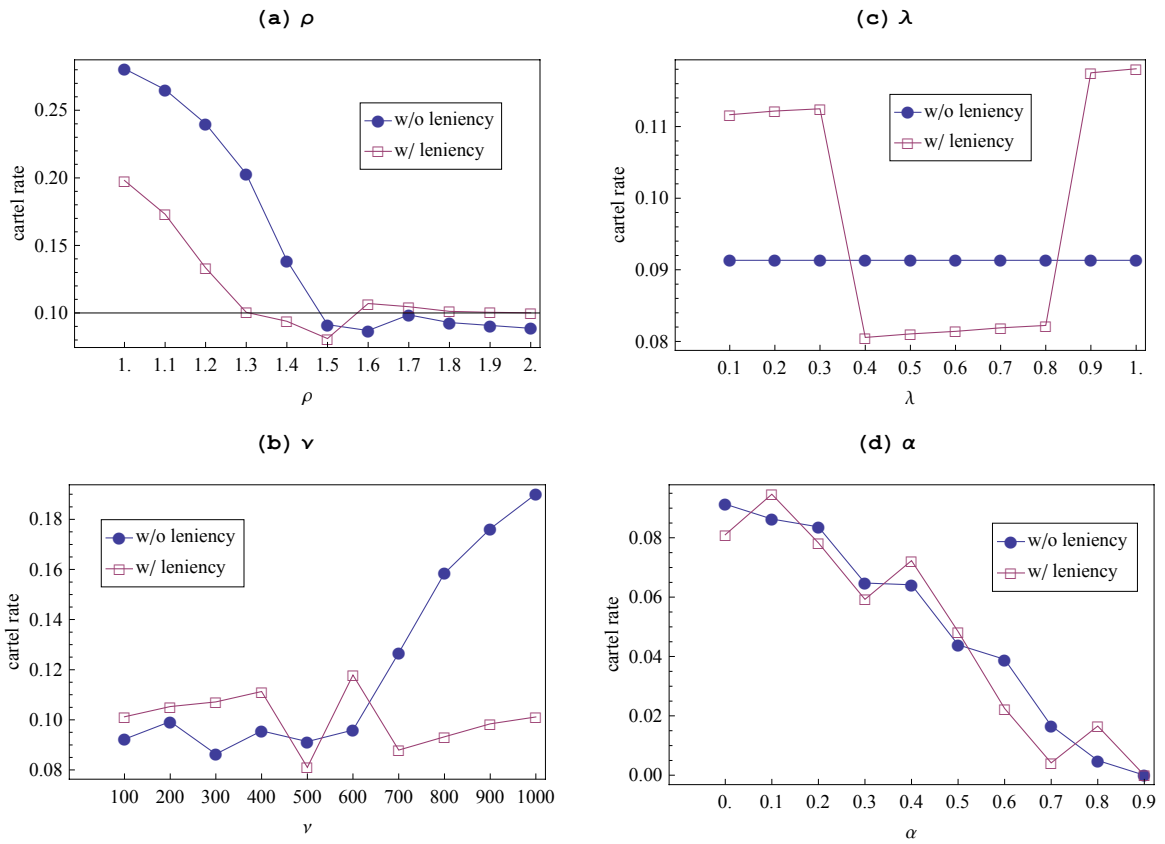


Figure 7 : Average Cartel Duration Conditional on η
(Baseline Case)

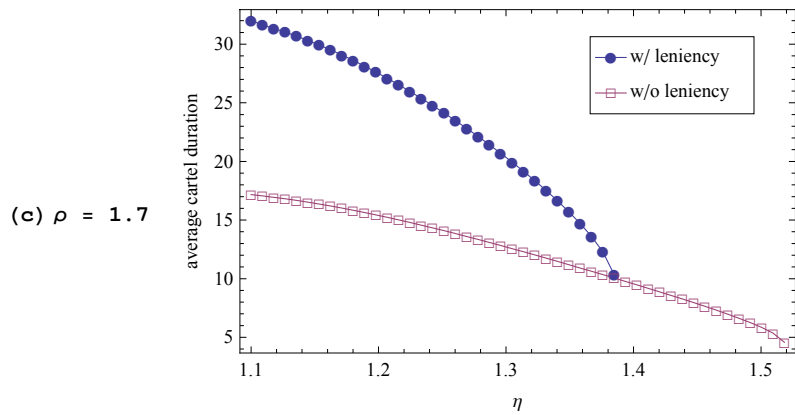
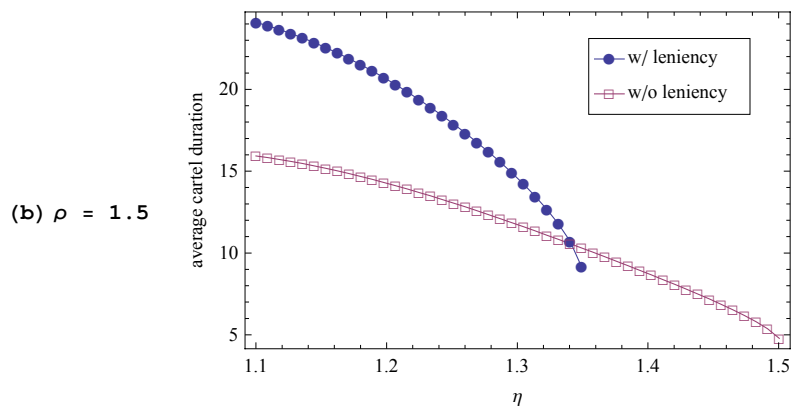
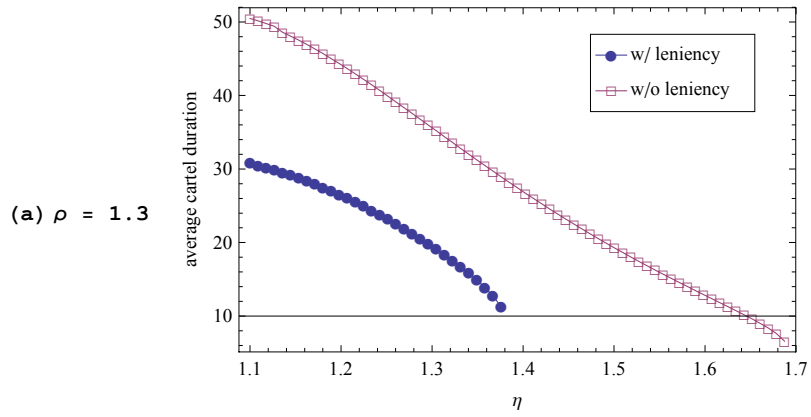


Figure 8 : Average Cartel Duration Conditional on η
for $\nu \in \{100, 1000\}$

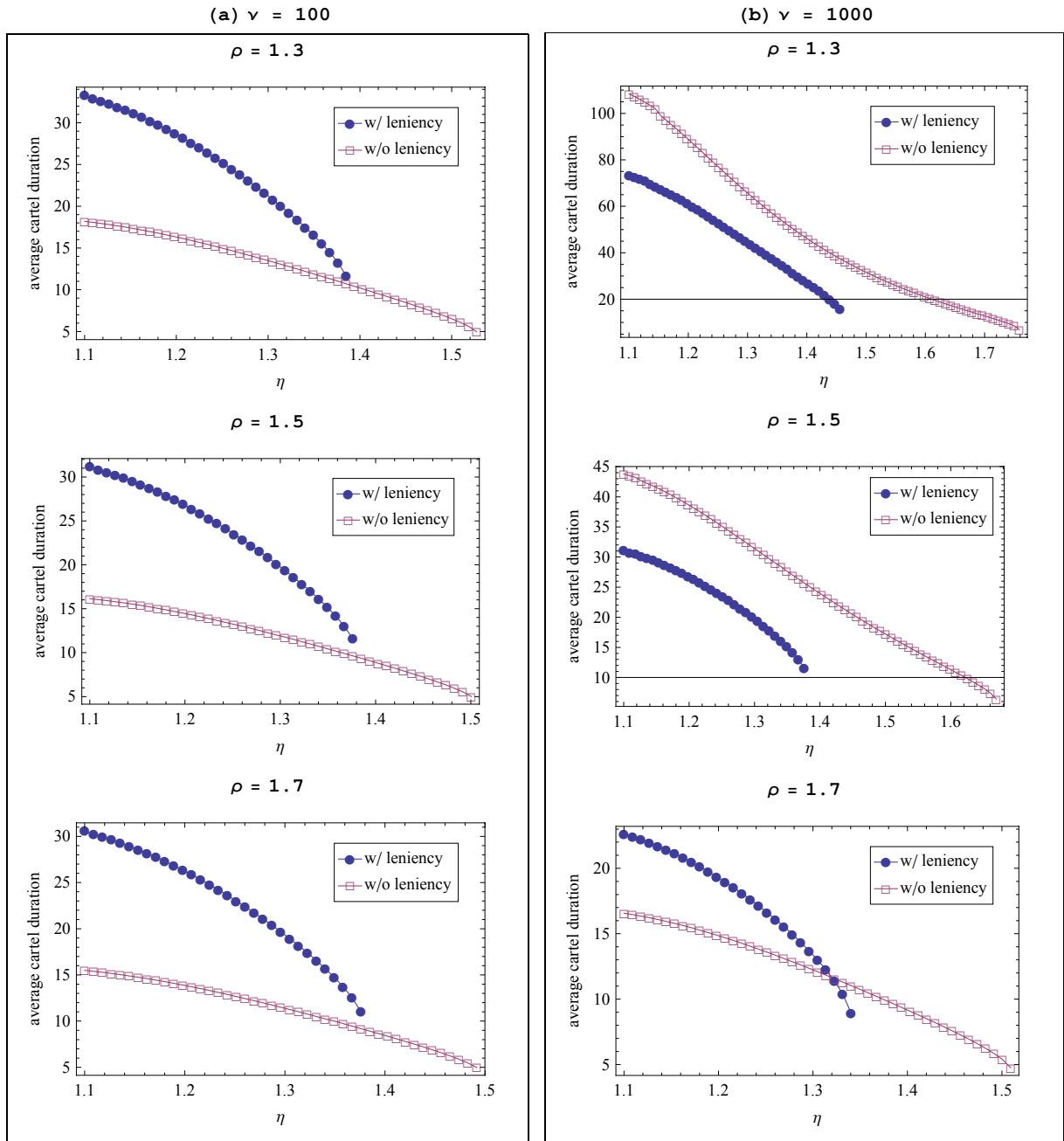


Figure 9 : Average Cartel Duration Conditional on η
for $\lambda \in \{0.2, 0.8\}$

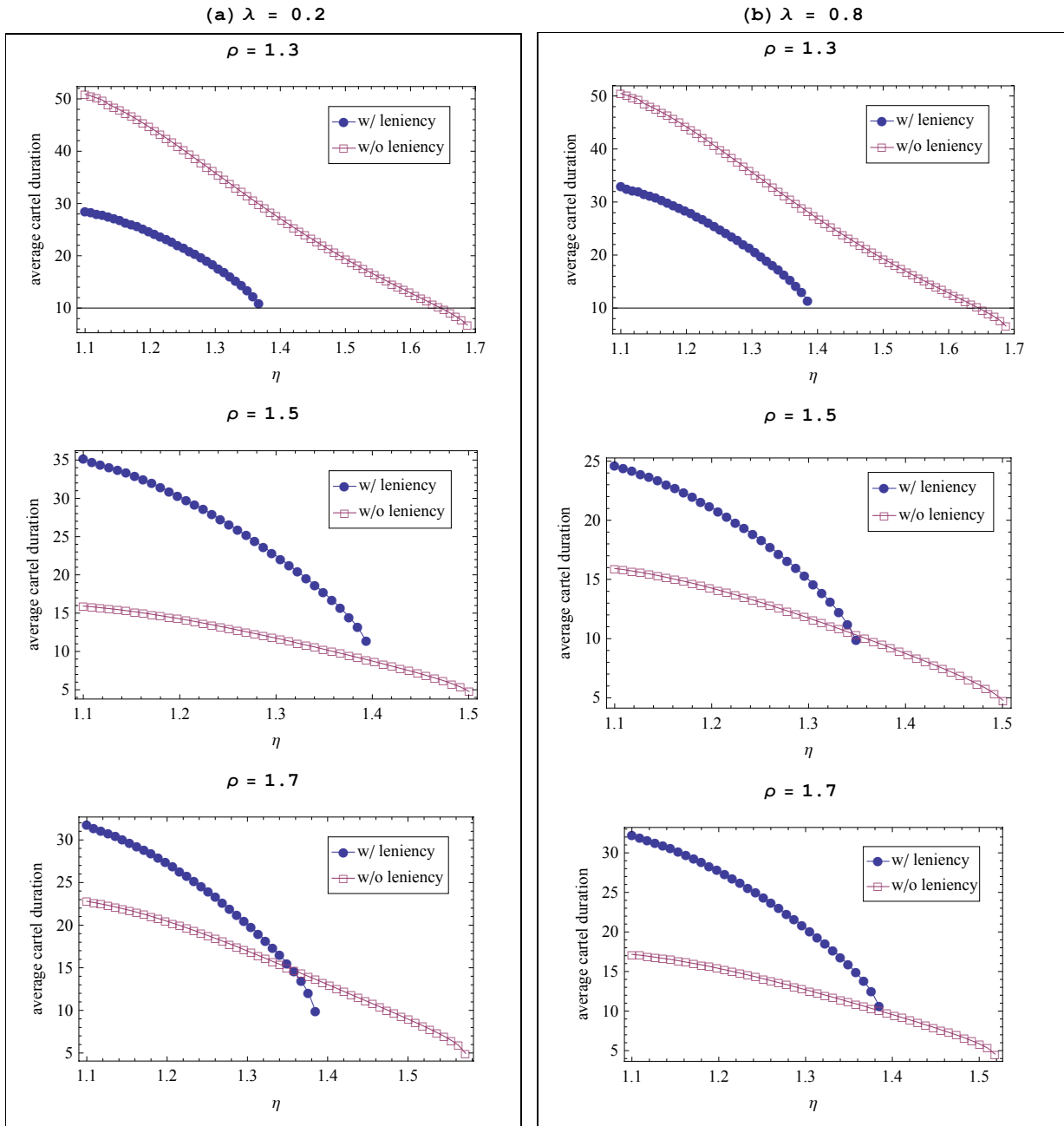


Figure 10 : Average Cartel Duration Conditional on η
for $\alpha \in \{0.2, 0.5\}$

