

THE INTERACTIVE EFFECT OF PRODUCT DIFFERENTIATION AND COST VARIABILITY ON PROFIT

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It is generally believed that industries with greater product differentiation have higher rates of return. This paper shows that this effect breaks down in the presence of firm-specific cost shocks. Greater substitutability in products generates two opposing effects: (1) it allows a larger increase in demand when a firm has a favorable cost shock, which more than compensates for the reduction in demand when it has an unfavorable cost shock, and (2) it results in more intense price competition. These two countervailing forces result in industry profit being highest in markets with a moderate degree of product differentiation.

1. INTRODUCTION

A well-established understanding is that greater product differentiation reduces the intensity of price competition and thereby generates higher firm and industry profit. An implication is that a firm should try to create considerable "distance" between its products and the products of its competitors.¹ In research involving industry analysis, the predicted positive relationship between product differentiation and profit has resulted in the inclusion of a proxy for product differentiation, typically the advertising-to-sales ratio, in structure-perfor-

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1. For example, Porter (1980, pp. 34-46).

mance studies.² The relationship between product differentiation and firm profitability is thus relevant to both managers making decisions and economists trying to understand industries.

Our analysis suggests a need to revise this understanding of how product differentiation influences profit. We consider a standard class of product differentiation models, which includes the Hotelling model as a special case. The only modeling innovation is to allow for firm-specific cost shocks. The introduction of such cost shocks causes average profit to be a nonmonotonic function of the degree of product differentiation. When products are highly differentiated, the average industry and firm profit can be raised by reducing the extent of product differentiation. Average profit is maximized at a moderate degree of differentiation.

The intuition behind this result can be traced back to the seminal paper of Oi (1961), which showed that a price-taking firm prefers price variability. Although in our setting it is a price-setting firm that faces cost variability, the same preference for variability exists. If the firm did not adjust its price in response to cost changes, its expected profit would be unaffected by cost variability, but if it optimally adjusts price, it can raise its expected profit by lowering its price when cost is low and raising its price when cost is high. It follows that the firm's expected profit is increasing in the variance of its cost shock.

Our analysis uncovers an interaction between the degree of product differentiation and the returns to cost variability. When products are highly substitutable, a firm that experiences a favorable cost shock can expect a large increase in demand, because its resulting low price will attract many of its competitor's customers. In comparison, when firms are local monopolies, the rise in a firm's demand when its cost (and price) is low is constrained by the boundaries of its own market. While it is also true that greater product substitutability means that a firm can expect a large decrease in demand when it has an unfavorable cost shock, the marginal gain in profit from having a bigger rise in demand when cost is low more than compensates for the bigger fall in demand when cost is high. As a result, average profit is higher with more substitutable products, given the presence of firm-specific cost variability. It follows that reducing product differentiation magnifies the returns to cost variability. Offsetting this effect is the intensification of price competition from having less-differentiated products. Balancing these two effects, industry profit is greatest in

2. See, for example, the discussion and review in Salinger (1990) and Weiss (1991).

markets characterized by a moderate degree of product differentiation.³

The paper is organized as follows. The basic model and the structure of the game are presented in Section 2. In Section 3, we analyze the effect of product differentiation on expected profits when firms' cost shocks are independent and thus firm-specific. We further assume that the cost shocks are private information. The initial assumptions on the nature of cost shocks are relaxed in Sections 4 and 5. Section 4 allows for industry-wide cost shocks by considering the case of correlated cost shocks. It is shown that our result is valid as long as the shocks are *less than perfectly* correlated. In Section 5, we turn to the possibility that the firms may be fully informed about each other's costs when choosing prices. Our result is shown to be robust to this specification as well. Section 6 discusses the implications of our analysis, and Section 7 concludes the paper.

2. THE MODEL

Let us consider a market that consists of two firms producing differentiated products. The demand function facing firm i is assumed to take the following form:

$$D_i(p_i, p_j) = \alpha - \beta p_i - \gamma(p_i - p_j), \quad (1)$$

where $\alpha, \beta, \gamma > 0$ and $i, j = 1, 2$ ($i \neq j$). Products are thus symmetrically differentiated, where γ measures the degree of their substitutability. A rise in γ is associated with a higher cross-price elasticity of demand between firms' products. When $\gamma = 0$, each firm has a local monopoly, while the case of perfectly homogeneous products is achieved as $\gamma \rightarrow +\infty$.⁴

The production technology entails constant returns to scale, where unit cost is subject to a stochastic shock:

$$C_i(q_i) = c_i q_i = (\hat{c}_i + \eta_i) q_i, \quad i = 1, 2. \quad (2)$$

3. When firms offer lines of products and consumers incur a cost to shopping at more than one firm, Klemperer (1992) finds that profit can be higher with less-differentiated products. The reason is that price competition is *less* intense with more substitutable products. In contrast, we find that prices are lower with less-differentiated products and, in spite of this fact, average profit can be higher.

4. This demand specification is used, for example, in Rotemberg and Saloner (1987) and Harrington (1995). When $\beta = 0$, eq. (1) is the demand structure derived from the Hotelling line model when firms are maximally differentiated and transport costs are linear. In that setting, $1/\gamma$ is the transport cost parameter.

\hat{c}_i is deterministic, while η_i is a cost shock observable only to firm i . In Sections 3 and 4 the cost shocks are assumed to be private information; in Section 5 we allow them to be public information. $\eta_i \in [\underline{\eta}_i, \bar{\eta}_i]$ and has a continuous cumulative distribution function denoted by $H_i(\cdot)$. We initially assume that η_1 and η_2 are independently and identically distributed with $E[\eta_i] = 0$ and $\text{Var}[\eta_i] = \sigma_{\eta_i}^2$. (In Section 4, we allow firms' cost shocks to be correlated and show that our central finding is robust.)

Each firm observes its cost shock, η_i . While the actual realization of η_i is assumed to be private information to firm i , $H_i(\cdot)$ is common knowledge. Once firms observe their own unit costs, they simultaneously choose prices. Given the demand and cost structures in eqs. (1) and (2), the realized profit of firm i is

$$\pi_i(p_i, p_j) = [\alpha - \beta p_i - \gamma(p_i - p_j)](p_i - c_i). \quad (3)$$

Firms are assumed to maximize expected profit.

3. FIRM-SPECIFIC COST SHOCKS

Since each firm chooses its price after observing its cost shock, a firm's strategy is a function that maps from the space of cost shocks into the space of prices. The uncertainty that a firm faces is over its rival's price, since it does not know its rival's cost, although it is presumed that it has an accurate conjecture of its rival's pricing strategy.

After observing its cost shock, the expected profit of firm i from choosing price p_i is

$$[\alpha - \beta p_i - \gamma(p_i - E_i p_j)](p_i - c_i). \quad (4)$$

$E_i p_j$ is firm i 's expectation of firm j 's price. It follows from eq. (4) that the first-order condition for firm i is

$$-(\beta + \gamma)(p_i - c_i) + \alpha - \beta p_i - \gamma(p_i - E_i p_j) = 0. \quad (5)$$

Taking firm j 's expectation over the above first-order condition yields

$$-(\beta + \gamma)(E_j p_i - \hat{c}_i) + \alpha - \beta E_j p_i - \gamma(E_j p_i - E_i p_j) = 0. \quad (6)$$

Rearranging eq. (6) for $E_i p_j$, we obtain

$$E_i p_j = \frac{2}{\gamma}(\beta + \gamma)E_j p_i - \frac{1}{\gamma}(\beta + \gamma)\hat{c}_i - \frac{\alpha}{\gamma}, \quad i, j = 1, 2 \quad (i \neq j). \quad (7)$$

Note that eq. (7) represents a pair of equations, one for firm 1 and one for firm 2. Solving these equations simultaneously for $E_i p_j$ and

$E_j p_i$ and substituting them into eq. (5), we derive the equilibrium price function for firm i :

$$p_i^*(c_i; \gamma) = \alpha(2\beta + \gamma)^{-1} + \frac{1}{2}c_i + \frac{1}{2}\gamma^2[4(\beta + \gamma)^2 - \gamma^2]^{-1}\hat{c}_i + \gamma(\beta + \gamma) \times [4(\beta + \gamma)^2 - \gamma^2]^{-1}\hat{c}_j. \quad (8)$$

Note that firm i 's equilibrium price depends on its realized cost, its expectation of firm j 's cost, and the ex ante expectation of its own cost. The last follows from the fact that firm j 's price depends on \hat{c}_i .⁵

In deriving these equilibrium price functions, it was implicitly assumed that an interior equilibrium was achieved for all cost shocks; that is,

$$\alpha - \beta p_i^*(\hat{c}_i + \eta_i) - \gamma[p_i^*(\hat{c}_i + \eta_i) - p_j^*(\hat{c}_j + \eta_j)] > 0 \quad \text{for all } (\eta_i, \eta_j) \in [\underline{\eta}_i, \bar{\eta}_i] \times [\underline{\eta}_j, \bar{\eta}_j]. \quad (9)$$

Restrictions on demand and cost parameters are assumed such that eq. (9) is true. Sufficient conditions for eq. (9) to be true are that $\hat{c}_1 + \bar{\eta}_1$ and $\hat{c}_2 + \bar{\eta}_2$ are sufficiently small and the cost differential, $|\hat{c}_1 + \eta_1 - \hat{c}_2 - \eta_2|$, is sufficiently small for all $(\eta_1, \eta_2) \in [\underline{\eta}_1, \bar{\eta}_1] \times [\underline{\eta}_2, \bar{\eta}_2]$.

Proposition 1 shows that as firms' products become more substitutable, price competition intensifies.

PROPOSITION 1: *The equilibrium price is lower as products are better substitutes: $\partial p_i^* / \partial \gamma < 0$, $i = 1, 2$.*

Proof. Differentiating eq. (8) with respect to γ , we obtain

$$\frac{\partial p_i^*}{\partial \gamma} = -(2\beta + 3\gamma)^2(\alpha - \beta\hat{c}_j) + 4\beta\gamma(\beta + \gamma)(\hat{c}_i - \hat{c}_j). \quad (10)$$

Since an interior equilibrium requires that $\alpha - \beta\hat{c}_j > 0$, the RHS of eq. (10) is unambiguously negative when $\hat{c}_i \leq \hat{c}_j$. To analyze the case of $\hat{c}_i > \hat{c}_j$, let us derive the necessary and sufficient condition for an interior equilibrium when $c_i = \hat{c}_i$ and $c_j = \hat{c}_j$. Substituting (\hat{c}_i, \hat{c}_j) into eq. (9), one derives

$$\alpha(2\beta + 3\gamma) - (2\beta^2 + 4\beta\gamma + \gamma^2)\hat{c}_i + \gamma(\beta + \gamma)\hat{c}_j > 0. \quad (11)$$

Rearranging the LHS of eq. (11), the following is obtained:

5. Note that $(p_1^*(\cdot), p_2^*(\cdot))$ is a Bayes-Nash equilibrium.

$$\begin{aligned}
 &-(2\beta + 3\gamma)^2(\alpha - \beta\hat{c}_j) + 4\beta\gamma(\beta + \gamma)(\hat{c}_i - \hat{c}_j) \\
 &< -4\beta\gamma(\beta + 2\gamma)(\hat{c}_i - \hat{c}_j) - (2\beta + 3\gamma)(2\beta^2 + \gamma^2)(\hat{c}_i - \hat{c}_j). \quad (12)
 \end{aligned}$$

The LHS of the inequality is the expression for $\partial p_i^*/\partial \gamma$. Since the RHS is less than zero when $\hat{c}_i > \hat{c}_j$, it must be true that $\partial p_i^*/\partial \gamma < 0$ when $\hat{c}_i > \hat{c}_j$. \square

Given $p_1^*(c_1)$ and $p_2^*(c_2)$ from eq. (8), it is straightforward to derive the equilibrium profit:

$$\begin{aligned}
 \pi_i^*(c_i) &= [\alpha - (\beta + \gamma)p_i^*(c_i) + \gamma E_i(p_j^*(c_j))][p_i^*(c_i) - c_i] \\
 &= (\beta + \gamma)\{\alpha(2\beta + \gamma)^{-1} - \frac{1}{2}c_i + \frac{1}{2}\gamma^2[4(\beta + \gamma)^2 - \gamma^2]^{-1}\hat{c}_i \\
 &\quad + \gamma(\beta + \gamma)[4(\beta + \gamma)^2 - \gamma^2]^{-1}\hat{c}_j\}. \quad (13)
 \end{aligned}$$

We are now prepared to address the following question: What is the relationship between the degree of product differentiation and industry profitability? Using eq. (13), firm i 's average profit is represented by its expected profit prior to the realization of its cost shock:

$$\begin{aligned}
 E_i[\pi_i^*(c_i)] &= \int_{\underline{\eta}_i}^{\bar{\eta}_i} \pi_i^*(\hat{c}_i + \eta_i) dH_i(\eta_i) \\
 &= \frac{1}{4}(\beta + \gamma)(2\beta + 3\gamma)^{-2}(2\beta + \gamma)^{-2} \\
 &\quad \times \{(2\beta + 3\gamma)[2\alpha - (2\beta + \gamma)\hat{c}_i] + \gamma[\gamma\hat{c}_i + 2(\beta + \gamma)\hat{c}_j]\}^2 \\
 &\quad + \frac{1}{4}(\beta + \gamma)\sigma_{\eta_i}^2. \quad (14)
 \end{aligned}$$

Consistent with the logic of Oi (1961), a firm's average profit is increasing in the variance of its cost shock. By optimally raising price when cost is high and lowering price when cost is low, a firm does better, on average, when there is variability in its cost function.

3.1 SYMMETRIC CASE

To begin, let us consider the symmetric case:

$$\hat{c}_1 = \hat{c}_2 = \hat{c}, \quad \sigma_{\eta_1}^2 = \sigma_{\eta_2}^2 = \sigma_{\eta}^2. \quad (15)$$

Given eq. (15), eq. (14) becomes

$$E_i[\pi_i^*] = (\beta + \gamma)(\alpha - \beta\hat{c})^2(2\beta + \gamma)^{-2} + \frac{1}{4}(\beta + \gamma)\sigma_{\eta}^2. \quad (16)$$

The following equations show the relationship between product differentiation and industry profit:

$$\frac{\partial E_i[\pi_i^*]}{\partial \gamma} = -\gamma(\alpha - \beta\hat{c})^2(2\beta + \gamma)^{-3} + \frac{1}{4}\sigma_\eta^2, \quad (17)$$

$$\frac{\partial^2 E_i[\pi_i^*]}{\partial \gamma^2} = 2(\alpha - \beta\hat{c})^2(\gamma - \beta)(2\beta + \gamma)^{-4}. \quad (18)$$

If there is no cost variability (that is, if $\sigma_\eta^2 = 0$), then $\partial E_i[\pi_i^*]/\partial \gamma < 0$ for all $\gamma > 0$. In other words, the average profit is lower in markets with less-differentiated products, *ceteris paribus*. However, when we allow for cost variability, eq. (17) shows that profit is increasing in γ for markets with highly differentiated products (that is, where γ is low). By eq. (18), average profit is concave in γ for $\gamma < \beta$ and convex in γ for $\gamma > \beta$. When the variance of cost shocks is positive but not too large, it follows from eqs. (17) and (18) that the average profit takes the form in Figure 1.⁶ Industry profit is maximized in industries with a moderate degree of product differentiation.

As product substitutability rises, the lower-priced firm's demand increases while the higher-priced firm's demand declines. Since a favorable cost shock typically translates into a lower price than one's rival's, greater product substitutability implies an even greater average demand increase after a favorable cost shock and an even greater average demand decrease (that is, demand falls more) after an unfavorable cost shock. Roughly speaking, a firm's demand gets shifted from when its cost is high to when its cost is low. Given that the marginal profit gain from more demand is greater as cost is lower, it follows that average profit is higher with more substitutable products (holding fixed the effect of product differentiation on the intensity of price competition). Offsetting this effect is that reduced product differentiation intensifies price competition (Proposition 1).

These two countervailing forces result in industry profit being maximized at a moderate degree of product differentiation. Markets

6. When $\sigma_\eta^2 > 0$ but sufficiently small, Figure 1 is an accurate depiction of average profit for all $\gamma \in [0, \beta + \theta]$ for some $\theta > 0$. For two reasons, it may not be an accurate depiction for large values of γ . Since average profit is a third-order polynomial in γ , it can turn from being decreasing to being increasing in γ when γ is sufficiently large. However, one must recognize that our expression is only relevant for sufficiently small values of γ . Recall that the existence of an interior equilibrium requires that the cost differential be sufficiently small. The higher is the value of γ , the lower must the maximal cost differential be in order to ensure that an interior equilibrium exists for all cost shocks. In particular, as $\gamma \rightarrow +\infty$, so that firms' products become perfect substitutes, the maximum cost differential for an interior equilibrium to always exist goes to zero. Given the distribution on cost shocks, this means that Figure 1 is an accurate characterization of the relationship between average profit and product substitutability when γ is not too great. The maximal value of γ for which Figure 1 is applicable is greater, the lower is the variance in the cost shocks. As the variance in the cost shocks goes to zero, the maximal value of γ to ensure an interior equilibrium goes to $+\infty$.

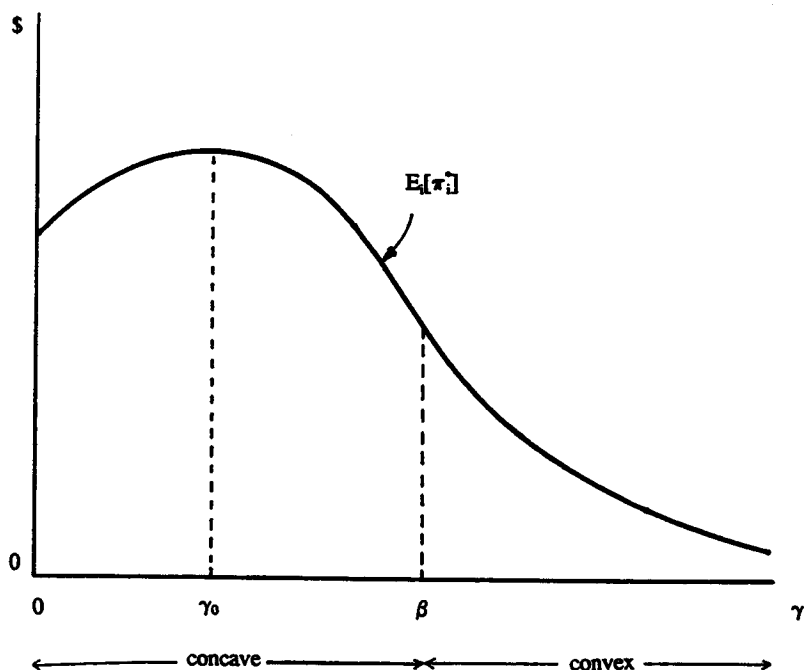


FIGURE 1.

in which firms have local monopolies (γ close to 0) minimize price competition but limit the gains to a favorable cost shock, because the resulting rise in demand is limited by the boundaries of one's own market. Markets with highly substitutable products allow for great gains in demand after a favorable cost shock (which is followed by a price reduction), but are plagued by intense price competition. Industries with a moderate degree of product differentiation allow for considerable returns to cost variability while keeping price competition in check.

3.2 ASYMMETRIC CASE

Our result is robust to allowing one firm to be more efficient. Using eq. (14), one can derive

$$\left. \frac{\partial E_i[\pi_i^*(c_i)]}{\partial \gamma} \right|_{\gamma=0} = \frac{1}{4\beta} (\alpha - \beta \hat{c}_i)(\hat{c}_j - \hat{c}_i) + \frac{1}{4} \sigma_{\pi_i}^2. \quad (19)$$

We are then evaluating the effect of changing the degree of product

differentiation on firms' average profits when products are maximally differentiated. Clearly, when $\hat{c}_i \leq \hat{c}_j$, so that firm i has lower cost on average, its average profit is increasing in the degree of product substitutability. This is not surprising, since firm i can generally expect to have a lower price. Given that it is the lower-priced firm that benefits from greater product substitutability, the more efficient firm's average profit is increasing in γ . Of course, even in this case, it will eventually be decreasing in γ , due to the intensification of price competition. More interesting is that even when a firm is commonly less efficient (i.e., $\hat{c}_i > \hat{c}_j$), its average profit is still greater when products are more substitutable, as long as the cost disadvantage is not too great.⁷

Even if there is a persistent difference in the efficiency of firms, firm and industry profit are not monotonically increasing in the degree of product differentiation. Profit is higher in industries characterized by a moderate degree of product differentiation than in markets where firms have local monopolies.

4. THE CASE OF CORRELATED COST SHOCKS

Our analysis up to this point has assumed that the cost shocks are independent of one another. This implies that the cost shocks are firm-specific, so that a firm's expectation of the rival's cost is in no way influenced by the realization of its own costs. In many situations, however, cost shocks are correlated across firms, as we will assume in this section. For the sake of tractability, we will assume that the random variables η_1 and η_2 are characterized by a *bivariate normal distribution* with correlation coefficient ρ , so that

$$E(\eta_i) = 0, \quad \text{var}(\eta_i) = \sigma_{\eta_i}^2 = \sigma_{\eta}^2 \quad \text{for } i = 1, 2, \quad \text{cov}(\eta_1, \eta_2) = \rho\sigma_{\eta}^2. \quad (20)$$

We maintain the demand and cost structures described in Section 2, except for the fact that the cost shocks are correlated. After observing its own cost shock, the expected profit of firm i from choosing price p_i is then

$$\{\alpha - \beta p_i - \gamma(p_i - E_i[p_j | c_i])\}(p_i - c_i). \quad (21)$$

7. In a deterministic case with zero variance, the sign of $\partial E_i[\pi_i^*(c_i)]/\partial \gamma |_{\gamma=0}$ in eq. (19) depends purely on the sign of $\hat{c}_j - \hat{c}_i$. In this instance, the less efficient firm would always prefer maximal differentiation. With positive variance, however, the less efficient firm's average profit may increase with higher γ if the cost disadvantage is sufficiently small.

Contrary to the previous case of independent cost shocks, firm i 's expectation of its rival's price is now conditional upon the realization of its own cost, c_i , or more specifically, η_i . This is due to the fact that firm j 's optimal price is a function of its cost, c_j , and firm i 's expectation of c_j is now conditional upon its own cost, c_i . It follows from maximization of eq. (21) with respect to p_i that firm i 's best response function is

$$\psi_i(E_i[p_j | c_i]) = \frac{1}{2} \alpha(\beta + \gamma)^{-1} + \frac{1}{2} c_i + \frac{1}{2} \gamma(\beta + \gamma)^{-1} E_i[p_j | c_i]. \quad (22)$$

In this section, we will use the method of Cyert and DeGroot (1970) in solving for the unique equilibrium price functions. Since firm i knows that firm j will choose a price equal to $\psi_j(E_j[p_i | c_j])$, $E_i[\psi_j(E_j[p_i | c_j]) | c_i]$ can be substituted for $E_i[p_j | c_i]$ in eq. (22). Infinitely iterating the substitution process in this manner, we generate the following expansion for firm i 's best response function:

$$\begin{aligned} \psi_i[p_j | c_i] &= \frac{1}{2} \alpha(\beta + \gamma)^{-1} + \frac{1}{2} c_i + \frac{1}{4} \alpha \gamma (\beta + \gamma)^{-2} \\ &\quad \times [B^0 + B^1 + B^2 + B^3 + \dots] \\ &\quad + \frac{1}{4} \gamma (\beta + \gamma)^{-1} \hat{c}_j [A^0 + A^1 + A^2 + A^3 + \dots] \\ &\quad + \frac{1}{8} \gamma^2 (\beta + \gamma)^{-2} \hat{c}_i [A^0 + A^1 + A^2 + A^3 + \dots] \\ &\quad + \frac{1}{4} \gamma (\beta + \gamma)^{-1} E_i[\eta_j | \eta_i] + \frac{1}{8} \gamma^2 (\beta + \gamma)^{-2} \\ &\quad \times E_i[E_j[\eta_i | \eta_j] | \eta_i] \\ &\quad + \frac{1}{16} \gamma^3 (\beta + \gamma)^{-3} E_i[E_j[E_i[\eta_j | \eta_i] | \eta_j] | \eta_i] \\ &\quad + \frac{1}{32} \gamma^4 (\beta + \gamma)^{-4} E_i[E_j[E_i[E_j[\eta_i | \eta_j] | \eta_i] | \eta_j] | \eta_i] + \dots, \end{aligned} \quad (23)$$

where $A = \frac{1}{4} \gamma^2 (\beta + \gamma)^{-2} \in (0, 1)$ and $B = \frac{1}{2} \gamma (\beta + \gamma)^{-1} \in (0, 1)$. Note that with correlated shocks the best response function contains a series of iterated conditional expectations. To simplify the expression for the best response function, we use the assumption that η_1 and η_2 have a bivariate normal distribution. The conditional expectation then takes the following simple form:⁸

8. When two random variables, x_1 and x_2 , have a bivariate normal distribution and their means and variances are $E(x_i) = \mu_i$ and $\text{Var}(x_i) = \sigma_i^2$, the conditional expectation of x_j given x_i is given by $E[x_j | x_i] = \mu_j + \rho(\sigma_j/\sigma_i)(x_i - \mu_i)$ for $i, j = 1, 2$, where ρ is the correlation coefficient. [See DeGroot (1975), p. 250.] Since in our case $\mu_1 = \mu_2 = 0$ and $\sigma_1 = \sigma_2 = \sigma_\eta$, eq. (24) follows immediately.

$$E_i[\eta_j | \eta_i] = \rho\eta_i. \quad (24)$$

Taking iterative conditional expectations over eq. (24), we derive the following series of expressions, which depend only on ρ and η_i :

$$\begin{aligned} E_i[E_j[\eta_i | \eta_j] | \eta_i] &= \rho^2\eta_i, \\ E_i[E_j[E_i[\eta_j | \eta_i] | \eta_j] | \eta_i] &= \rho^3\eta_i, \\ E_i[E_j[E_i[E_j[\eta_i | \eta_j] | \eta_i] | \eta_j] | \eta_i] &= \rho^4\eta_i, \\ &\vdots \\ &\vdots \\ &\vdots \end{aligned} \quad (25)$$

By replacing the iterated conditional expectations in eq. (23) with the above expressions, simplifying the infinite series, and simultaneously solving the best-response functions, we arrive at the equilibrium price function:

$$\begin{aligned} p_i^*(c_i; \gamma) &= \alpha(2\beta + \gamma)^{-1} + \frac{1}{2}c_i + \gamma(\beta + \gamma)(2\beta + 3\gamma)^{-1}(2\beta + \gamma)^{-1}\hat{c}_j \\ &\quad + \frac{1}{2}\gamma^2(2\beta + 3\gamma)^{-1}(2\beta + \gamma)^{-1}\hat{c}_i + \frac{1}{2}\gamma\rho[2(\beta + \gamma) - \gamma\rho]^{-1}\eta_i. \end{aligned} \quad (26)$$

It can be shown that $\partial p_i^*/\partial\gamma < 0$, $i = 1, 2$, as in Proposition 1. The equilibrium profit is now given by

$$\begin{aligned} \pi_i^*(c_i) &= (\beta + \gamma)\{\alpha(2\beta + \gamma)^{-1} + \frac{1}{2}c_i + \gamma(\beta + \gamma)(2\beta + 3\gamma)^{-1} \\ &\quad \times (2\beta + \gamma)^{-1}\hat{c}_j + \frac{1}{2}\gamma^2(2\beta + 3\gamma)^{-1}(2\beta + \gamma)^{-1}\hat{c}_i \\ &\quad + \frac{1}{2}\gamma\rho[2(\beta + \gamma) - \gamma\rho]^{-1}\eta_i\}^2. \end{aligned} \quad (27)$$

Using eq. (27), one can derive firm i 's expected profit prior to the realization of its cost shock:

$$\begin{aligned} E_i[\pi_i^*(c_i)] &= (\beta + \gamma)(2\beta + 3\gamma)^{-2}(2\beta + \gamma)^{-2} \\ &\quad \times [\alpha(2\beta + 3\gamma) - (2\beta^2 + 4\beta\gamma + \gamma^2)\hat{c}_i + \gamma(\beta + \gamma)\hat{c}_j]^2 \\ &\quad + (\beta + \gamma)[2(\beta + \gamma) - \gamma\rho]^{-2}[(\beta + \gamma) - \gamma\rho]^2\sigma_{\eta_i}^2. \end{aligned} \quad (28)$$

Differentiating the expected profit with respect to γ and evaluating it at $\gamma = 0$, we obtain

$$\left. \frac{\partial E_i[\pi_i^*(c_i)]}{\partial\gamma} \right|_{\gamma=0} = \frac{1}{4\beta}(\alpha - \beta\hat{c}_i)(\hat{c}_j - \hat{c}_i) + \frac{1}{4}(1 - \rho)\sigma_{\eta_i}^2. \quad (29)$$

When costs are symmetric, so that $\hat{c}_1 = \hat{c}_2$, the average profit is unambiguously increasing in γ at $\gamma = 0$, as long as cost shocks are not perfectly correlated (i.e., for $\rho < 1$). Furthermore, the rate of increase in average profit is larger as the correlation coefficient ρ is smaller. If the correlation between the cost shocks is less than perfect, average industry profit will be higher in those industries characterized by moderate product differentiation than in the industries with local near-monopolies.

For asymmetric costs ($\hat{c}_1 \neq \hat{c}_2$) with imperfectly correlated shocks ($\rho < 1$), our previous results, obtained under the assumption of independent shocks, continue to be valid. When firm i has a lower cost on average, i.e., $\hat{c}_i < \hat{c}_j$, its average profit is increasing in the degree of product substitutability. Even when firm i has a cost disadvantage on average, so that $\hat{c}_i > \hat{c}_j$, its average profit is still greater when products are more substitutable, as long as the cost differential is not too large. The maximum cost differential, which supports the increasing nature of the firm's average profit in product substitutability, depends on the degree of correlation, ρ . As ρ increases, this maximum cost differential tends to decline. When $\rho = 1$, so that the shocks are perfectly correlated, it is only the more efficient firm whose average profit increases in product substitutability at $\gamma = 0$.⁹

5. COST SHOCKS AS PUBLIC INFORMATION

In previous sections, we assumed that each firm's cost shock was private information. The implication of this assumption is that each firm, having observed its own cost shock, faces uncertainty over the rival's price due to the lack of information regarding the rival's cost. While this may be an accurate description for many industries, there are also situations in which firms are perfectly informed about each other's cost shocks. The realization of exchange-rate shocks, for instance, is known to all firms. We next allow for this possibility.

After observing the cost shocks, the profit of firm i from choosing price p_i is

$$[\alpha - \beta p_i - \gamma(p_i - p_j)](p_i - c_i). \quad (30)$$

9. A referee suggested the possibility that ρ may be a function of γ . It can be shown that our result is robust to this extension. Let us first examine the expression for expected profits in eq. (28). The only terms containing ρ in the expression are the interaction terms, $\gamma\rho$. With the assumption of $\rho = \rho(\gamma)$, those interaction terms become $\gamma\rho(\gamma)$. When these terms are differentiated with respect to γ and further evaluated at $\gamma = 0$, we obtain $\partial[\gamma\rho(\gamma)]/\partial\gamma|_{\gamma=0} = \gamma \partial\rho/\partial\gamma + \rho(\gamma)|_{\gamma=0} = \rho(0)$. Hence, with $\rho = \rho(\gamma)$ the only change that occurs in eq. (29) is that ρ is replaced with $\rho(0)$.

Deriving the firms' best response functions from eq. (30) and then simultaneously solving them, we obtain the equilibrium prices:

$$p_i^*(c_i, c_j) = (2\beta + \gamma)^{-1}[\alpha + (\beta + \gamma)c_i] \\ + (2\beta + 3\gamma)^{-1}(2\beta + \gamma)^{-1}\gamma(\beta + \gamma)(c_j - c_i). \quad (31)$$

Substituting eq. (31) into eq. (30), the equilibrium profit is found to be

$$\pi_i^*(c_i, c_j) = [(2\beta + 3\gamma)(2\beta + \gamma)]^{-2}(\beta + \gamma) \\ \times [(2\beta + 3\gamma)(\alpha - \beta c_i) + \gamma(\beta + \gamma)(c_j - c_i)]^2. \quad (32)$$

Firm i 's expected profit prior to observing its own cost and the rival's cost is then

$$E_i[\pi_i^*] = \int_{\underline{x}}^{\bar{\eta}} \int_{\underline{x}}^{\bar{\eta}} \pi_i^*(\hat{c} + \eta_i, \hat{c} + \eta_j) dH_i(\eta_i) dH_j(\eta_j) \\ = (2\beta + \gamma)^{-2}(\beta + \gamma)(\alpha - \beta\hat{c})^2 \\ + [(2\beta + 3\gamma)(2\beta + \gamma)]^{-2}(\beta + \gamma) \\ \times \{[\beta(2\beta + 3\gamma) + \gamma(\beta + \gamma)]^2 + \gamma^2(\beta + \gamma)^2\}\sigma_{\eta}^2. \quad (33)$$

Using eq. (33), we can show the following properties:

$$\frac{\partial E_i[\pi_i^*]}{\partial \gamma} \Big|_{\sigma_{\eta}^2=0} = -(2\beta + \gamma)^{-3}(\alpha - \beta\hat{c})^2 \gamma < 0 \quad \text{for all } \gamma > 0, \quad (34)$$

$$\frac{\partial E_i[\pi_i^*]}{\partial \gamma} \Big|_{\gamma=0} = \frac{1}{4} \sigma_{\eta}^2 > 0 \quad \text{for } \sigma_{\eta}^2 > 0. \quad (35)$$

When there is no cost variability ($\sigma_{\eta}^2 = 0$), the expected profit is strictly declining in the degree of product substitutability, as expected. When instead there exists cost variability ($\sigma_{\eta}^2 > 0$), the expected profit evaluated at $\gamma = 0$ (the point of maximal differentiation) is strictly increasing in γ , as we found in Section 3.

While it would be most desirable to have a full characterization of how the expected profit in eq. (33) depends on γ , the complexity of its expression prevents us from achieving that goal. Instead, we simulate the expression in eq. (33) for some parameter values. Figure 2 offers a graphical representation of the simulated expected profit functions in terms of γ for various values of σ_{η}^2 , given $\alpha = 200$, $\beta = 20$, and $\hat{c} = 5$. As can be seen, the existence of cost variability gives rise to expected profit being maximized at a moderate degree of product

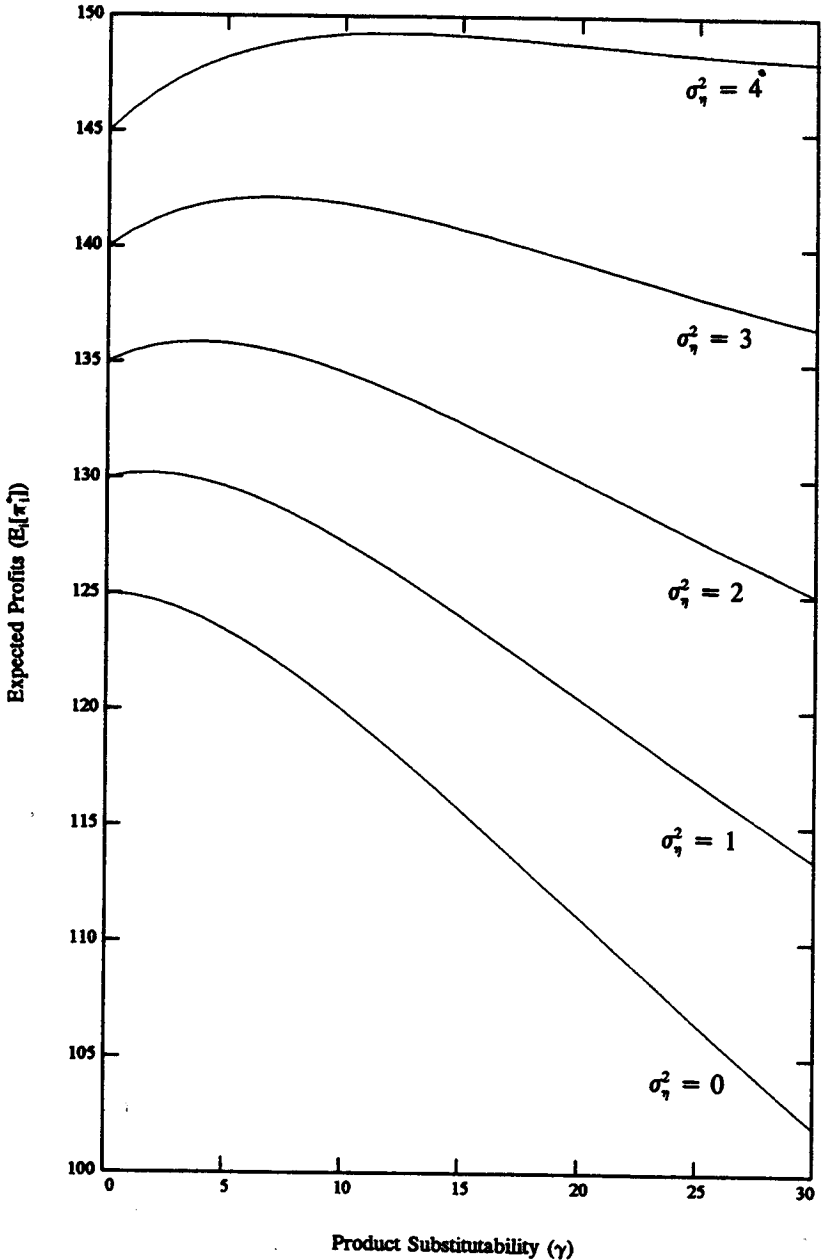


FIGURE 2. EXPECTED-PROFIT FUNCTIONS FOR VARIOUS σ_η^2 VALUES WHEN COST SHOCKS ARE PUBLIC INFORMATION ($\alpha = 200$, $\beta = 20$, $\ell = 5$)

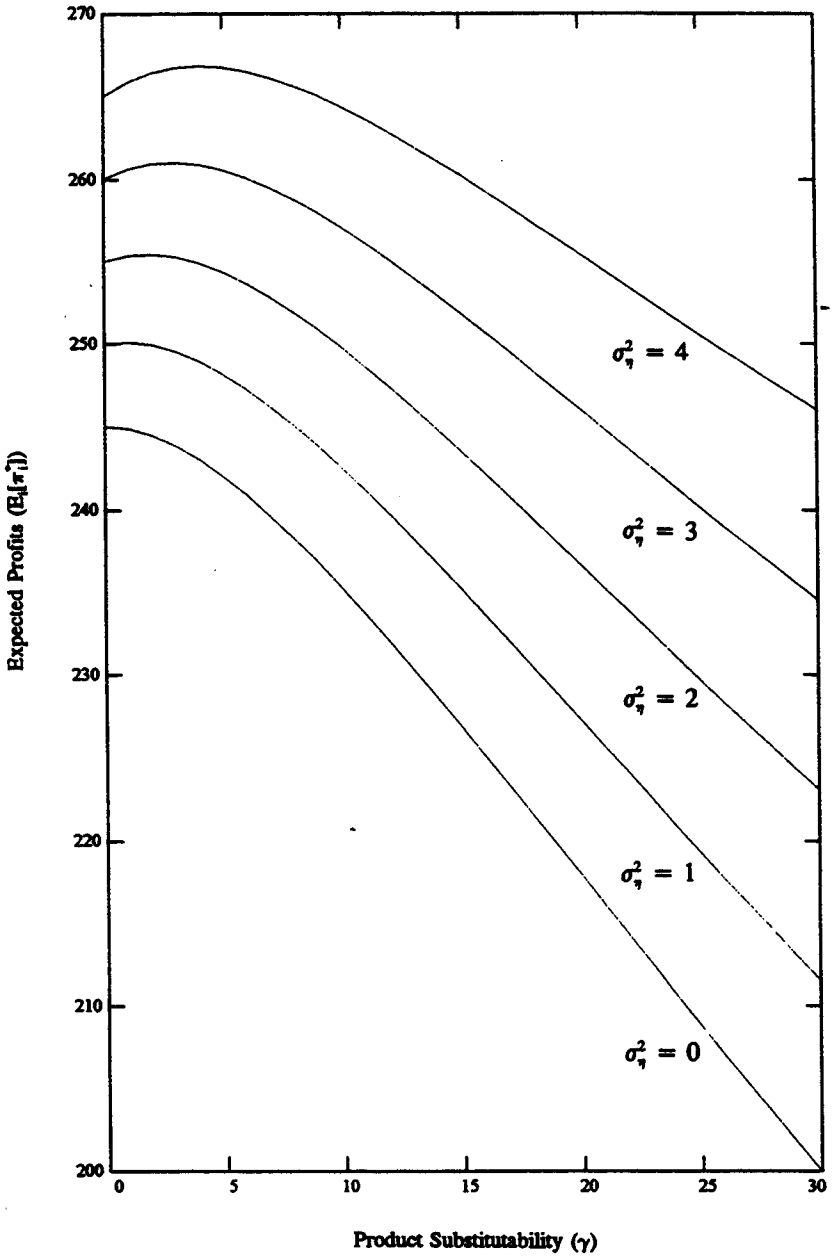


FIGURE 3. EXPECTED-PROFIT FUNCTIONS FOR VARIOUS σ_η^2 VALUES WHEN COST SHOCKS ARE PUBLIC INFORMATION ($\alpha = 240$, $\beta = 20$, $\ell = 5$)

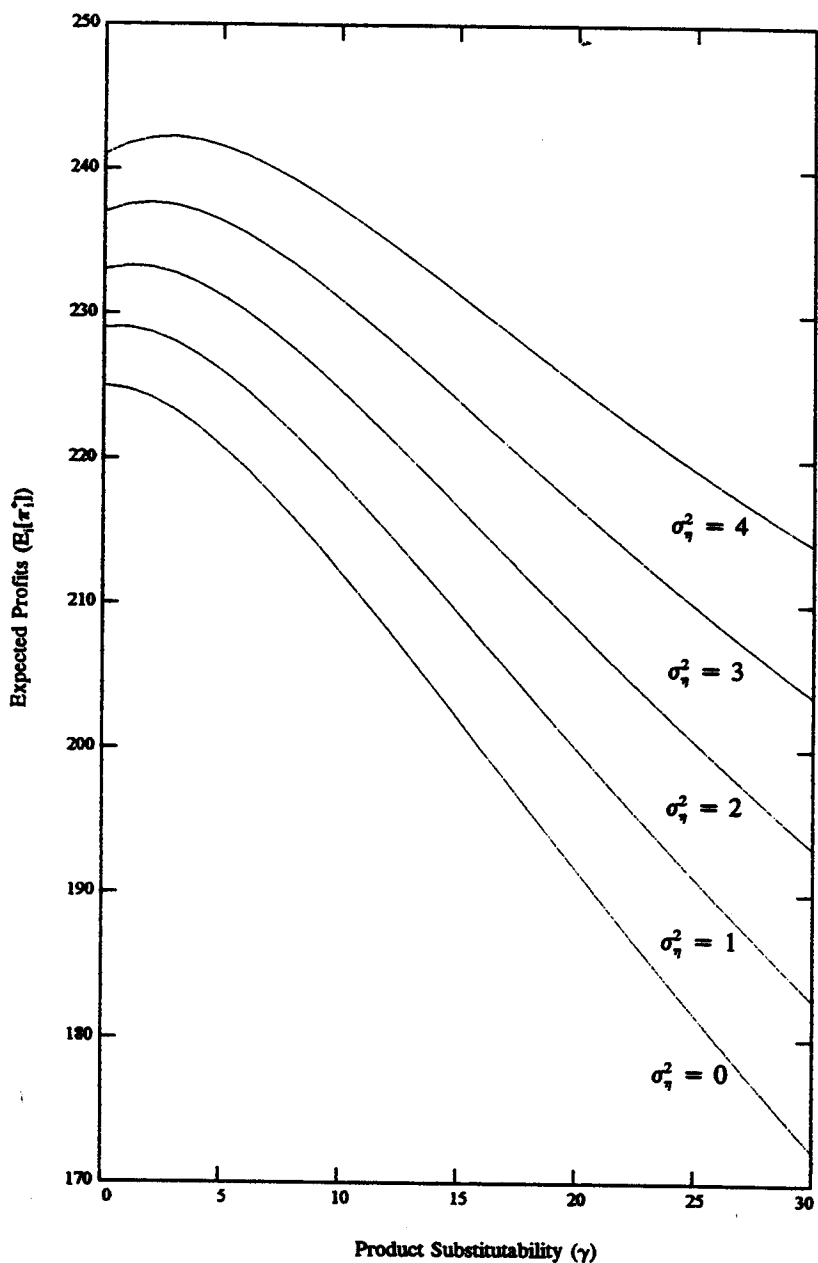


FIGURE 4. EXPECTED-PROFIT FUNCTIONS FOR VARIOUS σ_γ^2 VALUES WHEN COST SHOCKS ARE PUBLIC INFORMATION ($\alpha = 200$, $\beta = 16$, $\hat{c} = 5$)

differentiation. The simulation in Figure 2 is further enhanced by evaluating expected profit at various levels of α and β . In Figures 3 and 4, we retain $\hat{c} = 5$, while increasing the value of α to 240 and lowering the value of β to 16, respectively. In both of these cases, we find the expected profit function to be increasing in γ at $\gamma = 0$ as long as σ_n^2 is positive. These simulation results suggest that our central finding is robust to the assumption that the cost shocks may be public information.

6. DISCUSSION

The preceding analysis established that the presence of firm-specific cost shocks changes the qualitative relationship between product differentiation and firm and industry profit. The empirical relevance of our analysis hinges upon the size of firm-specific cost shocks. If variability in the cost differential between firms is small, then the effect we have identified is apt to be second-order.

We know of no studies of how variable firms' costs are, or how correlated their costs. While awaiting the empirical research, however, we suggest two settings in which variability in the cost differential between firms is likely to be significant. First, exchange-rate fluctuations represent a significant source of movements in the cost differential between a domestic firm and a foreign firm. Second, there is typically *ex ante* uncertainty over the cost of new entrants to a market. This implies uncertainty over the cost differential between either two entrants (in a completely new market) or between an entrant and an established firm. Thus, our conclusions may be most important for global and innovative markets.

6.1 GLOBAL MARKETS

One substantive source of variability in the differential in firms' costs is movements in the exchange rate of countries' currencies. For example, the relative cost advantage of Japanese auto manufacturers vis-à-vis American auto manufacturers in the U.S. market has fluctuated considerably over time as the yen has strengthened and weakened vis-à-vis the dollar. More generally, in markets containing domestic and foreign firms, we believe that the cost variability modeled in this paper is likely to be significant. Managers operating in global markets should take into account the implications of exchange-rate variability for their decisions about product design and location.

Our analysis also has implications for trade policy. Consider a potentially global market that is not realized because trade barriers

prevent each country's firms from entering the other countries' domestic markets. The act of opening up these markets to foreign competitors increases the substitutability of their products, since the initial situation was one of local monopolies—products were not substitutable, because one country's consumers were prevented from buying foreign products. Since our analysis showed that increasing the substitutability of firms' products can raise the average profit, it suggests that all firms, not just the consumers, could benefit from the elimination of trade barriers. This is true even if the domestic markets are identical and some countries' firms are known to have a more efficient production technology.

6.2 NEW AND GROWING MARKETS

As it pertains to our analysis, the relevant feature of either a new or a growing market is entry. The crucial feature of entry is that there is typically uncertainty over the cost of a new firm. In a new market, firms are apt to be uncertain as to who is more efficient in delivering the good to consumers.¹⁰ In an established market, a new firm will often enter with an improved but unproven production technology, while established firms have an advantage emanating from experience. It is then unclear who will have the cost advantage. The presence of this uncertainty over the cost differential makes our analysis relevant for markets experiencing entry. In this class of settings, cost variability should be interpreted as preentry uncertainty over the cost differential of firms. Of particular note is that our results may have implications for new firms' product choice or store location decisions. For example, a retailer might want to locate a new store close to an existing store because the upside is greater by doing so. If the retailer turns out to have a cost advantage and thereby charges lower prices, its profit will be much higher, as its lower prices will capture a larger share of its rival's customers than if it had located far away.

7. CONCLUDING REMARKS

It is generally believed that industries with greater product differentiation tend to have higher rates of return. In this paper, we have shown that the presumed monotonic relationship between product differentiation and industry profit breaks down in the presence of firm-specific cost shocks. Greater substitutability in firms' products raises the re-

10. For a competitive industry, Jovanovic (1982) explores the implications of firm uncertainty over who is more efficient.

turn to cost variability, by allowing a larger increase in demand when a firm has a favorable cost shock, which more than compensates for the reduction in demand when it has an unfavorable cost shock. The downside is that reduced product differentiation results in more intense price competition. These two countervailing forces result in industry profit being highest in markets characterized by a moderate degree of product differentiation.

Though this result was derived using specific functional forms, the intuition is general. What allows for the possibility that average profit is decreasing in product differentiation is that, holding prices constant, reducing product differentiation increases the demand of the low-cost firm, and a firm's average profit is raised by shifting demand from when its cost is high to when its cost is low.

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