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The effects of product differentiation on collusive pricing

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Using the spatial competition framework of Hotelling (1929), this paper explores the relationship between the degree of product differentiation and the ability of firms to collude with respect to price. We show that the minimum discount factor required to support the joint profit maximum as an equilibrium outcome monotonically increases as products become better substitutes. When the joint profit maximum cannot be supported, the optimal collusive price is shown to decline as products become more substitutable. These findings suggest that firms producing stronger substitutes tend to find it tougher to collude in terms of their product price.

1. Introduction

The traditional industrial organization literature has noted that product differentiation is one of the primary characteristics of market structure which affects the conduct and performance of firms in a market. The complexities in the nature of such a relationship have been emphasized by Bain (1968, page 330):

> The principal relevant dimensions of market structure would seem to be the degree of seller concentration in the industry, the condition of entry, and *the degree of product differentiation.* The main alternative conduct patterns are complete collusion, incomplete collusion of several varieties, and interdependent action without collusion. In general, almost any of these conduct patterns might theoretically be expected to be associated with any market structure outside the atomistic range.

Indeed, the task of determining the relationship between a particular type of

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market stucture and its consequent firm conduct has never been a trivial matter.

In a differentiated-products market, the pricing decision of a firm depends on the substitutability of rival firms' products because a firm's ability to command a high price for its product is severely limited when there exists an alternative product which is a good substitute. Quite plausibly, firms offering similar products would then have a strong incentive to coordinate their pricing decisions in order to avoid severe price competition. However, the temptation to defect from the collusive agreement is also strong if there is strong substitutability between the products. Defection is tempting because a slight reduction in price will result in a significant increase in firm demand: the elasticity of firm demand is greater, the better are substitutes. As a consequence, the effect of product substitutability on the firms' ability to sustain collusive pricing is far from obvious. The main objective of this paper is to examine the relationship between the degree of product differentiation and the ability of firms to collude with respect to price.

The issue of collusive pricing behavior in differentiated-products industry has only been mildly touched upon in past work; specifically Deneckere (1983). Using multi-product demand functions in the repeated game setting, he derives a trigger strategy equilibrium for both a price game and an output game when firms have differentiated products. In his model, the degree of product differentiation is determined by an exogenously given substitutability parameter embedded in the multi-product demand functions. Ease of collusion is measured by the minimum discount factor that supports the joint profit maximum as a subgame perfect equilibrium in the infinitely repeated game. In the price-setting repeated game Deneckere (1983) finds a non-monotonic relationship between the substitutability of products and the ability of firms to collude (as implied by the minimum discount factor); specifically, that collusion is more difficult when products are moderate substitutes than when they are very strong or very weak substitutes.[1]

In this paper, we take the address model approach to product differentiation by assuming consumers with heterogeneous tastes. This is in contrast to the multi-product demand function approach of Deneckere (1983), which assumes a representative consumer with a desire for variety. Our purpose to taking this formulation of product differentiation is to examine the robustness and sources of Deneckere's (1983) results, and gain further insight into how different models of product differentiation affect the analysis of collusive pricing behavior. The address model employed in our paper involves the spatial competition framework of Hotelling (1929) as modified by d'Aspremont et al. (1979) and Neven (1985).

[1]See Majerus (1988), Martin (1989) and Wernerfelt (1989) for extensions of Deneckere's (1983) model.

To summarize our results, we find that collusion is more difficult to sustain the smaller is the degree of product differentiation. In particular, the minimum discount factor supporting the joint profit maximum as an equilibrium outcome is found to monotonically increase as products become better substitutes. This result is in contrast to the non-monotonicity result of Deneckere (1983). Our finding suggests that firms producing stronger substitutes tend to find it tougher to collude in terms of their product price.

The organizing of this paper is as follows. The model of differentiated products is discussed in section 2. In section 3, we provide a formal description of the duopoly supergame. In order to aid our exposition of the repeated game, some results regarding the one-shot price game are briefly discussed in section 4. In section 5, we analyze the repeated game and present our main theorem. Comparative statics are performed in section 6 with regards to the effect of product substitutability on collusive pricing.

2. The model of differentiated products

We use the standard Hotelling (1929) market for differentiated products as modelled in Neven (1985). The differentiated commodity is represented in the product space X which is the unit interval $[0,1]$. There are two firms, each producing only one product at constant (zero) marginal cost. The location of firm i is denoted by $x_i \in X$, where firms are numbered so that $x_1 \leqq x_2$. Let Ω be the set of all possible pairs of product locations: $\Omega = \{(x_1, x_2) | x_1 \in [0,1], x_2 \in [0,1], x_1 \leqq x_2\}$. We also define a set C as a subset of Ω such that it contains only symmetric product location pairs, i.e., $C \equiv \{(x, 1-x) | x \in [0, 1/2)\}$.[2] Our investigation is mainly concerned with symmetric location pairs.

Consumers are uniformly distributed over $[0,1]$ where the location of a consumer represents his most preferred product. If consumer x^* purchases product $\hat{x}$ at price $p_{\hat{x}}$, his total cost is $p_{\hat{x}} + f(\hat{x}, x^*)$ where $f(\hat{x}, x^*)$ is the utility cost (in dollars) of purchasing a product different from the most preferred variety of x^*. Following the formulation of d'Aspremont et al. (1979), we assume the function to be quadratic, $f(\hat{x}, x^*) = b(\hat{x} - x^*)$.[3] Each consumer has a finite reservation price, k, so that she will buy one (zero) unit of the differentiated product if the delivered price is less than or equal to (greater than) k.[4] We specify k to be finite but sufficiently high. In

[2]Notice that the set C does not include $(x, 1-x) = (1/2, 1/2)$. When $x = 1/2$, the products are perfect substitutes. Since we are only concerned with 'differentiated' products, the case of homogeneous products is intentionally excluded from our analysis.

[3]The appeal of this assumption is that a pure-strategy Nash equilibrium has been shown to exist in prices for all locations by d'Aspremont et al. (1979) and Neven (1985).

[4]While the standard assumption is that consumer demand is perfectly inelastic at one unit, it results in the industry profit function being unbounded, thereby creating an analytical problem when firms collude. Therefore, we assume a finite consumer reservation price as was specified in Economides (1984).

particular, it is assumed that $(5/4)b \leqq k < \infty$. While finiteness of k is necessary to bound the strategy sets of the firms, $k \geqq (5/4)b$ is necessary for the entire market, $[0,1]$, to be served in equilibrium when firms do not collude. This will be discussed more fully later.

In this paper, we take the product locations to be exogenous to the model, where the relative locations signify a given degree of differentiation: Closely located products are interpreted by the consumers as being strongly substitutable. This paper is thus an initial step toward a more general framework in which firms are allowed to choose product as well as price. While a distinct advantage of address models over other models is its ability to endogenize product choice, we chose to remain in the exogenous location setting in order to explore the effect of product differentiation on collusive pricing. In addition, an analysis of this case is required prior to allowing for firms to choose their products. An endogenization of product choice in an intertemporal setting is pursued in Chang (1990).

3. Structure of the game

At the outset of the game, firms are located at the product location pair, (x_1, x_2). Given this location pair, firms engage in an infinitely repeated price game in which, in each period, firms simultaneously choose price, p_i^t, where $p_i^t \in [0,k]$ and $k < \infty$, $i = 1,2$. A price strategy of a firm is then an infinite sequence of action functions, $\{S_i^t\}_{t=1}^{\infty}$, where S_i^t: $[0,k]^{2x(t-1)} \to [0,k]$. We restrict firms to using pure strategies.

In any period t, we denote by $\hat{z}_t$ the consumer who is indifferent between purchasing x_1 and x_2 given prices (p_1^t, p_2^t). $\hat{z}_t$ is defined by the following relationship: $p_1^t + b(\hat{z}_t - x_1)^2 = p_2^t + b(\hat{z}_t - x_2)^2$. Demand for x_1 in period t is then defined by $\bar{z}(p_1^t, p_2^t; x_1, x_2)$ as follows:

$$\bar{z}(p_1^t, p_2^t; x_1, x_2) = \begin{cases} 0 & \text{if } \hat{z}_t < 0, \\ \hat{z}_t & \text{if } 0 \leqq \hat{z}_t \leqq 1, \\ 1 & \text{if } \hat{z}_t > 1. \end{cases} \tag{1}$$

Demand for x_2 is simply $[1 - \bar{z}(p_1^t, p_2^t; x_1, x_2)]$. The demand curve faced by a firm in this model is thus piecewise linear: for a given price of the rival it contains a segment in which two firms coexist and a segment in which a firm monopolizes the market by sufficiently undercutting its rival's price. This demand curve is quite comparable to that of Deneckere's (1983) which also exhibits piecewise linearity under similar conditions. The difference between the two demand curves is that the segment of our demand curve where the firm monopolizes the market is perfectly inelastic, where as that of Deneckere's (1983) demand curve is less than perfectly inelastic – quantity is

still a decreasing function of price even when the firm is a monopolist. However, this distinction will prove to be unimportant in the analysis.

Period t profits of firm i are then defined as follows:

$$\pi_1(p_1^t, p_2^t) = p_1^t \bar{z}(p_1^t, p_2^t, x_1, x_2). \tag{2}$$

$$\pi_2(p_1^t, p_2^t) = p_2^t(1 - \bar{z}(p_1^t, p_2^t, x_1, x_2)), \qquad t = 1, 2, \ldots, \infty. \tag{3}$$

Firms have a common discount factor, δ, and adopt a strategy vector $\sigma = (\sigma_1, \sigma_2)$, where $\sigma_i = \{S_i^1, S_i^2, \ldots, S_i^t, \ldots\}$ is the strategy of firm i. In any time period τ, firm i maximizes the discounted present value of its future profits starting from τ: $\sum_{t=\tau}^{\infty} \delta^{t-\tau} \pi_i(S_1^t, S_2^t)$, $i = 1, 2$. The equilibrium concept is that of subgame perfect equilibrium [Selten (1975)]. This concept requires firms' strategies to form a Nash equilibrium in all periods, $t = 1, 2, \ldots, \infty$, and for all histories. Incredible threats are thus ruled out.

4. One-shot price game

We start our analysis by investigating the equilibrium behavior of firms in the one-shot price game given product locations. Our objective is to provide us with the best response functions, Nash equilibrium prices, and the joint profit maximizing prices necessary for analyzing the repeated game. Since the time subscript may be ignored in the one-shot game, p_1 and p_2 denote the pair of prices charged by firms 1 and 2, respectively. Given the firm profits as defined by (2) and (3), it is straightforward to explicitly derive the best response price of firm i, $\psi_i(p_j)$, for any given price $p_j \in [0, k]$ of firm j:

$$\psi_1(p_2) = \begin{cases} (1/2)p_2 - (1/2)b(x_1^2 - x_2^2) & \text{if } p_2 < b(x_1^2 - x_2^2) + 4b(x_2 - x_1), \\ p_2 - b(x_1^2 - x_2^2) - 2b(x_2 - x_1) & \text{if } p_2 \geqq b(x_1^2 - x_2^2) + 4b(x_2 - x_1). \end{cases} \tag{4}$$

$$\psi_2(p_1) = \begin{cases} (1/2)p_1 + (1/2)b(x_1^2 - x_2^2) + b(x_2 - x_1) & \text{if } p_1 < -b(x_1^2 - x_2^2) + 2b(x_2 - x_1), \\ p_1 + b(x_1^2 - x_2^2) & \text{if } p_1 \geqq -b(x_1^2 - x_2^2) + 2b(x_2 - x_1). \end{cases} \tag{5}$$

It is important to note that it may be optimal for firm i to charge a

sufficiently low price and drive its opponent out of business if its rival's price is relatively high. When this is the case, the best response price of firm i entails charging the highest price that keeps its opponent out of the market. A similar situation arises in Deneckere's (1983) model, where a firm's best response price is the highest price that monopolizes the market. In both models, whether the optimal strategy is monopolization or market sharing, only the upper portion of the piece-wise linear demand curve is thus relevant to our analysis. This is because the kink price is the best response price for the firm whose optimal strategy is to monopolize the market. The lower portion of the demand curve, where the two models differ, is strategically irrelevant for the firm.

From (4) and (5), it is straightforward to solve for the Nash equilibrium prices, $(\bar{p}_1(x_1, x_2), \bar{p}_2(x_1, x_2))$, where $\bar{p}_1(x_1, x_2) = (2b/3)(x_2 - x_1) + (b/3)(x_2^2 - x_1^2)$, and $\bar{p}_2(x_1, x_2) = (4b/3)(x_2 - x_1) - (b/3)x_2^2 - x_1^2)$ – [Neven (1985)]. For the symmetrically located products $(x, 1-x) \in C$, it follows that $\bar{p}_1(x, 1-x) = \bar{p}_2(x, 1-x) = b(1-2x) \equiv \bar{p}(x)$. Note that the Nash equilibrium price is a declining function of x. Since the product locations are symmetric, a higher value of x implies a stronger degree of substitutability between products. Thus, we observe that when firms are competing in terms of price, the severity of price competition is increasing in the degree of product substitutability. This severe price competition tends to impose downward pressure on the equilibrium profits for the firms.[5]

Recall that we had initially required $k \geqq (5/4)b$. It is now easy to see why this condition is necessary and sufficient for the market to be fully supplied in equilibrium. Since $\bar{p}(x)$ reaches its maximum, b, at $x = 0$, it is the marginal consumer at 1/2 who pays the highest total cost. In order for the entire market to be served in Nash equilibrium, it is then necessary and sufficient that the total cost of this marginal consumer be less than or equal to the reservation price: $k \geqq (5/4)b$.

The next proposition presents a pair of prices that maximizes the joint profits for symmetrically located firms. Denote by $\hat{\pi}(p_1, p_2)$ the joint profit function of the cartel: $\hat{\pi}(p_1, p_2) = \pi_1(p_1, p_2) + \pi_2(p_1, p_2)$.

Proposition 1. For all symmetric locations, $(x, 1-x) \in C$, there exists a price, $p^m(x)$, such that $\hat{\pi}(p^m(x), p^m(x)) = \max_{p_1, p_2} \hat{\pi}(p_1, p_2)$, where

$$p^m(x) = \begin{cases} k - b[(1/2) - x]^2 & \forall x \in [0, 1/4), \\ k - bx^2 & \forall x \in [1/4, 1/2). \end{cases}$$

[5]If firms were to choose their product locations in anticipation of competition in the price game, we would expect them to sufficiently differentiate their products in order to avoid the severity of price competition. Indeed, d'Aspremont et al. (1979) and Neven (1985) found that the unique subgame perfect equilibrium for the two-stage game entails maximal differentiation of products.

Proof. See appendix.

Proposition 1 tells us that when products are symmetrically located, the joint profit maximizing price vector entails identical prices. It is worthwhile to note that $\partial p^m(x)/\partial x > 0$ for all $x \in [0, 1/4)$ and $\partial p^m(x)/\partial x < 0$ for all $x \in [1/4, 1/2)$.[6] Furthermore, it is observed that the entire market is served at a price of $p^m(x)$. Since consumers are uniformly distributed over the interval, $[0, 1]$, identical prices with symmetric locations imply equal market shares. Thus, joint profits are equally divided among the firms: $\pi_i(p^m(x), p^m(x)) = (1/2)p^m(x)$.

5. Trigger strategy equilibrium in the repeated game

In this section, we consider a model in which firms are given the product locations exogenously in the beginning of the horizon, and then engage in an infinitely repeated price game thereafter. Our objective is to examine the effects of product differentiation on the sustainability of collusive pricing among firms. In supporting collusion, the firms are assumed to use the 'grim trigger strategy' of Friedman (1971).

For ease of analysis, we assume products are located symmetrically at $(x, 1-x)$. Given these locations, firms try to support some collusive price, $p \in [\bar{p}(x), p^m(x)]$. Note that p is identical for both firms. Since the products are located symmetrically and the joint profit maximizing price, $p^m(x)$, is symmetric in the one-shot game, the optimal path of the collusive price in the long run would be symmetric and stationary as well. Therefore, if the firms adhere to the collusive agreement, each firm earns the collusive payoff, $\pi_i(p, p)$, in every period that they collude, where $\pi_i(p, p) = (1/2)p$.

$$S_i^1 = p; \tag{6}$$

$$S_i^t = \begin{cases} p & \text{if } p_j^\tau = p,\ \tau = 1, 2, \ldots, t-1,\ j = 1, 2; \\ \bar{p}(x) & \text{otherwise;} \quad t = 2, \ldots, \infty, \quad i = 1, 2. \end{cases}$$

In period 1 of the game, firms set price at the collusive level, p. In any period $t > 1$, firms will continue to charge p, as long as both firms have adhered to the agreement in the past. If either firm deviates from p, both firms immediately and permanently revert to the static Nash equilibrium price, $\bar{p}(x)$, as described in section 4. If a firm decides to deviate in any period, it would choose its best response price given that the other firm prices at the collusive level, p. Therefore, the deviation price of firm i will be $\psi_i(p)$ as

[6] Joint profits are thus highest when $(x, 1-x) = (1/4, 3/4)$.

defined in (4) and (5), and accordingly, its deviation payoff is $\pi_i(\psi_i(p), p) + [\delta/(1-\delta)]\pi_i(\bar{p}(x), p(x))$.

In order to support a collusive outcome with a credible threat of reverting to the one-shot Nash equilibrium, it must be shown that the strategies in (6) form a subgame perfect equilibrium. First, consider the history which involves a past defection so that firms are currently at the one-shot Nash equilibrium. Since the repetition of the one-shot Nash equilibrium forms a subgame perfect equilibrium, it is trivial to support (6) as a subgame perfect equilibrium for this set of histories. Alternatively, the history may involve collusive behavior throughout the course of the game. In this case, (6) would be an equilibrium if and only if each firm finds it more profitable to collude than to deviate:

$$L(p,\delta) \equiv [\delta/(1-\delta)][\pi_i(p,p) - \pi_i(\bar{p}(x), \bar{p}(x))] \geqq \pi_i(\psi_i(p), p) - \pi_i(p,p)$$

$$\equiv R(p). \tag{7}$$

When (7) is satisfied, the sum of the discounted future losses, $L(p,\delta)$, outweighs the one-time gain from optimally defecting, $R(p)$, and neither firm has an incentive to defect.

Denote by $\tilde{p}(\delta; x) \in [\bar{p}(x), p^m(x)]$, the best collusive price, which is defined as follows:

$$\pi_i(\tilde{p}(\delta;x), \tilde{p}(\delta;x)) = \max_p \pi_i(p,p) \quad \text{s.t.} \quad p \in \{p | L(p,\delta) \geqq R(p)\}. \tag{8}$$

$\tilde{p}(\delta, x)$ is the price that maximizes joint profits under the constraint that the strategies in (6) form a subgame perfect equilibrium.

Theorem 1. For all $(x, 1-x) \in C$, *there exists a critical discount factor,* $\hat{\delta}(x) < 1$, *such that*

$$\tilde{p}(\delta, x) = \begin{cases} p^m(x) & \forall \delta \geqq \hat{\delta}(x) \\ b(1-2x)\mu(\delta) & \forall \delta < \hat{\delta}(x) \end{cases}$$

$$\text{where} \quad \mu(\delta) = \begin{cases} (1+3\delta)/(1-\delta) & \text{for } \delta \in [0, 1/3], \\ (2-3\delta)/(1-2\delta) & \text{for } \delta \in (1/3, 1/2). \end{cases}$$

Proof. Let us first consider $p = p^m(x)$. From (7), we know that (6) is a subgame perfect equilibrium if and only if $L(p^m(x), \delta) \geqq R(p^m(x))$. Notice

that $L(p^m(x), \delta)$ is monotonically increasing in δ, while $R(p^m(x))$ is independent of δ. Furthermore, we observe that $L(p^m(x), 0) = 0 < R(p^m(x) < \lim_{\delta \to 1} L(p^m(x), \delta) = \infty$. Hence, there exists $\hat{\delta}(x)$ such that $L(p^m(x), \delta) \geqq R(p^m(x))$ if and only if $\delta \geqq \hat{\delta}(x)$, which automatically implies that $\tilde{p}(\delta; x) = p^m(x)$ if and only if $\delta \geqq \hat{\delta}(x)$.

For $\delta < \hat{\delta}(x)$, we must consider some $p \in [\bar{p}(x), p^m(x))$, since $p^m(x)$ can not be supported. Notice that $L(\bar{p}(x), \delta) = 0 = R(\bar{p}(x))$, and $\lim_{p \to p^m(x)} L(p, \delta) < \lim_{p \to p^m(x)} R(p) \forall \delta < \hat{\delta}(x)$. Since the joint profits are strictly increasing in p, we can see that $\tilde{p}(\delta; x) = \max \{p \in [\bar{p}(x), p^m(x)) | L(p, \delta) \geqq R(p)\}$. It is then immediate that $\tilde{p}(\delta; x)$ satisfies $L(\tilde{p}(\delta; x), \delta) = R(\tilde{p}(\delta; x))$, where

$$L(\tilde{p}(\delta; x), \delta) \equiv [\delta/(1-\delta)][\pi_i(\tilde{p}(\delta; x), \tilde{p}(\delta; x)) - \pi_i(\bar{p}(x), \bar{p}(x))]$$

$$= [\delta/(1-\delta)][(1/2)\tilde{p}(\delta; x) - (1/2)b(1-2x)], \tag{9}$$

$$R(\tilde{p}(\delta; x)) \equiv \pi_i(\psi_i(\tilde{p}(\delta; x)), \tilde{p}(\delta; x)) - \pi_i(\tilde{p}(\delta; x), \tilde{p}(\delta; x))$$

$$= \begin{cases} [1/8b(1-2x)][\tilde{p}(\delta; x) + b(1-2x)]^2 - (1/2)\tilde{p}(\delta; x) & \forall \delta \in [0, 1/3] \\ (1/2)\tilde{p}(\delta; x) - b(1-2x) & \forall \delta \in (1/3, 1/2). \end{cases} \tag{10}$$

Equating $L(\tilde{p}(\delta; x), \delta) = R(\tilde{p}(\delta; x))$ and solving for $\tilde{p}(\delta; x)$, we obtain the unique $\tilde{p}(\delta; x)$ for a given $\delta < \hat{\delta}(x)$. *Q.E.D.*

Since $\bar{p}(x) = b(1-2x)$, the collusive price $\tilde{p}(\delta; x)$ remains above the Nash equilibrium price $\bar{p}(x)$ for all $x < 1/2$.

6. Comparative static results

The purpose of this section is to fully characterize the critical discount factor, $\hat{\delta}(x)$, and perform comparative statics on the best collusive price, $\tilde{p}(\delta; x)$. Recalling the non-monotonicity result of Deneckere (1983), let us proceed to examine $\hat{\delta}(x)$ in our model.

From Theorem 1, we know that $L(p^m(x), \hat{\delta}(x)) = R(p^m(x))$. By substituting in the relevant payoff functions, $\hat{\delta}(x)$ can be derived as follows: For $x \in [0, 1/4)$,

$$\hat{\delta}(x)=\begin{cases}\dfrac{k-b[(1/2)-x]^2-b(1-2x)}{k-b[(1/2)-x]^2+3b(1-2x)} & \text{if } k-b[(1/2)-x]^2 \leqq 3b(1-2x), \\ \dfrac{k-b[(1/2)-x]^2-2b(1-2x)}{2k-2b[(1/2)-x]^2+3b(1-2x)} & \text{if } k-b[(1/2)-x]^2 > 3b(1-2x).\end{cases} \tag{11}$$

For $x \in [1/4, 1/2)$,

$$\hat{\delta}(x)=\begin{cases}\dfrac{k-bx^2-b(1-2x)}{k-bx^2+3b(1-2x)} & \text{if } k-bx^2 \leqq 3b(1-2x), \\ \dfrac{k-bx^2-2b(1-2x)}{2(k-bx^2)-3b(1-2x)} & \text{if } k-bx^2 > 3b(1-2x).\end{cases} \tag{12}$$

Property 1. $\partial\hat{\delta}(x)/\partial x > 0 \quad \forall x \in [0, 1/2)$.

Recall from section 4 that, with symmetric locations, the higher is x, the better substitutes they are. Our result then implies that collusion is more difficult as the products become better substitutes. Since this result is not immediately obvious from our model, it is essential that we understand the intuition behind it. When the products are horizontally differentiated, the degree of product differentiation is observed to have two countervailing effects on the ability of firms to collude. The first is that the greater the substitutability, the greater the gain from colluding. This is due to the fact that price competition is more severe when the products are more substitutable, and is reflected in that $\tilde{p}(x)=b(1-2x)$ is a decreasing function of x. Second, the better substitutes goods are, the greater is the one-time gain from defection. A defector needs only to offer a slightly lower price in order to capture the entire market when the substitutability among goods is greater. Our result suggests that the latter effect dominates, so that collusion is more difficult the smaller is the degree of product differentiation.

Notice that this result differs from that of Deneckere (1983). While we find a monotonic relationship between the substitutability of products and the ability of firms to collude, Deneckere (1983) found that collusion is more difficult when products are moderate substitutes than when they are very strong or very weak substitutes. The discrepancy in our results can be attributed to the way in which product differentiation is modelled. Figs. 1 and 2 show the general shapes of the single period collusive payoff and defection payoff in Deneckere's (1983) model and my model, respectively, as functions of product substitutability. The collusive payoff in Deneckere's (1983) model is strictly declining in product substitutability because consumer demand falls monotonically as products become more substitutable.

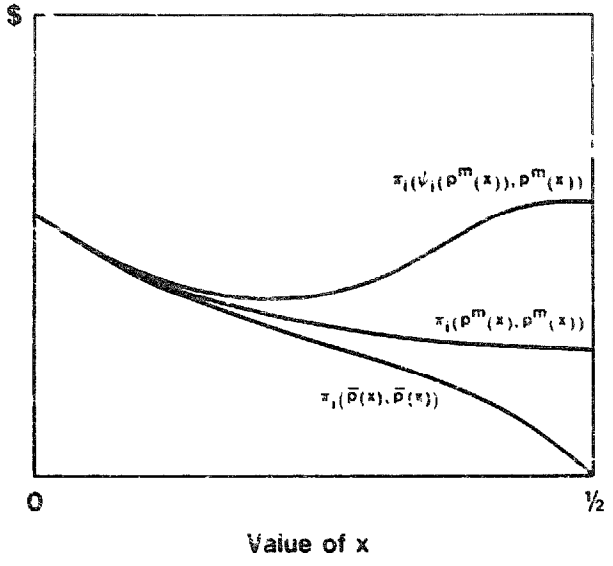

Fig. 1. Defection payoff and collusive payoff in Deneckere model.

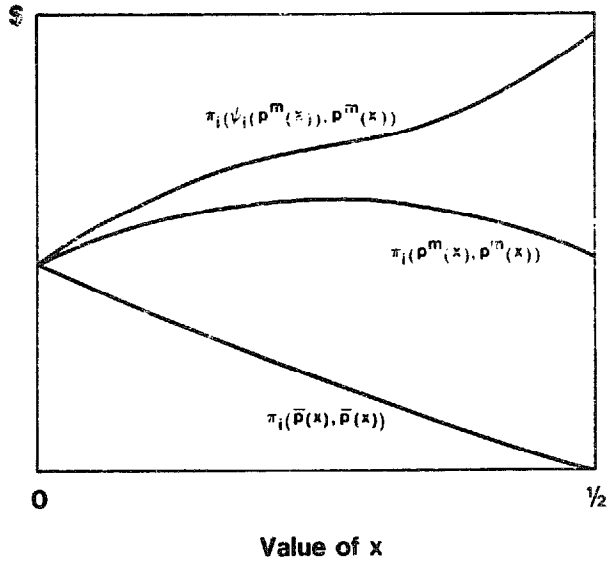

Fig. 2 Defection payoff and collusive payoff in location model.

This result arises naturally in the multi-product demand function model of product differentiation, where the linear inverse demand takes the following form: $p_i=\alpha_i-\beta_i q_i-\gamma q_j$, $i, j=1, 2$.[7] $\gamma>0$ represents the substitutability

[7]This demand system can be inverted to $q_i(p_i, p_j)$ as long as $q_i>0$ and $q_j>0$.

between products i and j. Since the representative consumer prefers variety, his(her) demand declines as products become more and more similar. On the other hand, our model of product differentiation produces a collusive payoff that is non-monotonic and achieves a maximum at $x = 1/4$. While the payoff is non-monotonic, it remains high for the entire range of substitutability. When the products are differentiated so as to satisfy consumers with heterogeneous tastes, it is optimal for the colluding firms to serve all consumers by appropriately adjusting the collusive price. Since consumers are willing to purchase the product up to the reservation price, and the entire market is served at the collusive price, it is easily seen that the collusive payoff can be maintained at a high level.

As can be seen in figs. 1 and 2, the defection payoffs in the two models also differ in their shapes. The defection payoff in Deneckere's (1983) model declines for weaker substitutes, but starts to increase as products become stronger substitutes. Fig. 2 indicates that our model results in a defection payoff that is monotonically increasing in the degree of product substitutability. Let us first look at the region of weaker substitutes, where we observe the discrepancy in the movement of defection payoffs between the two models. In this region, the defecting firm prefers to share the market with its rival, and the shape of defection payoff in each model is very dependent upon the shape of collusive payoff previously discussed. In this region of weaker substitutes, our model entails a collusive payoff (and collusive price) that is rising with greater substitutability due to the fact that the degree of differentiation approaches that of social optimum. The defection payoff thus rises along with the collusive payoff. In Deneckere's (1983) model, however, the collusive payoff is declining monotonically in this region because of consumer's preference for diversity. The defection payoff declines with greater substitutability following the movement of the collusive payoff.

Next, we look at the region of stronger substitutes, where both models generate rising defection payoffs. Note that in this region the defecting firm optimally monopolizes the market by undercutting its rival. In the region where the best defection strategy is to monopolize the market, product substitutability is observed to have two opposing effects on defection payoff. First, for a given (fixed) collusive price in this region, the optimal defection price (monopolizing the market) is higher for stronger substitutes, thus providing a higher defection payoff. Second, the collusive *price* (charged by the rival firm) itself may be lower for stronger substitutes lowering the payoff to the defector. The final outcome is then dependent upon the relative strength of these two opposing effects. While both effects exist in our model, the first effect dominates the second effect giving rise to the defection payoff rising with greater substitutability. In Deneckere's (1983) model, the second effect has no impact on the final outcome since his collusive price is independent of product substitutability. Thus, Deneckere's defection payoff

also rises with greater substitutability when undercutting so as to capture the entire market is the optimal defection strategy.

The non-monotonicity of the minimum discount factor in Deneckere (1983) appears to be due to the non-monotonic defection payoff.[8] In particular, for strong substitutes the defection payoff rises with greater substitutability at a decreasing rate, while the Nash equilibrium payoff falls at an increasing rate. Thus, as products become very strong substitutes, the punishment in the form of price competition becomes much more severe making the defection strategy increasingly less attractive. Conversely, in our model it is the increasing attractiveness of the gains from defection for stronger substitutes that dominates the fear of price competition. Collusion is thus more difficult to sustain when products are stronger substitutes.

Property 2. $\partial\hat{\delta}(x)/\partial b < 0 \qquad \forall x \in [0, 1/2)$.

For any product location, it becomes increasingly more difficult to support collusion as the transport cost parameter, b, declines. The underlying reason for this result is that the incentive to cheat increases as transport cost becomes smaller; the firms can gain larger market share by undercutting its opponent's cartel price. This increased incentive to cheat makes collusion more difficult to support for all product locations. It is worthwhile to note that as $b \to 0$, the products approach perfect substitutes. No matter how much product differentiation we may have (in terms of location), the critical discount factor sustaining collusion would approach that for the case of homogeneous products as $b \to 0$.

The next two results provide properties of the best collusive price derived in Theorem 1.

Property 3. $\partial\tilde{p}(\delta; x)/\partial\delta > 0 \qquad \forall \delta \in (0, \hat{\delta}(x))$.

The best collusive price increases as the firm discount factor rises. This result is in line with one's intuition in that as firms value future profits more, cheating becomes relatively less attractive. Thus, the increased degree of cartel sustainability takes the form of a higher supportable collusive price.

By monotonicity of $\hat{\delta}(x)$ and Theorem 1, it is straightforward to show the following property:

Property 4.

There exists $x^0 < 1/2$ such that

[8]It should be noted that this result of Deneckere's (1983) is restricted to the case of duopoly price competition. The minimum discount factor *is* monotonic if firms compete in quantities [Deneckere (1983)]. Furthermore, it can be shown that even when firms compete in terms of price, the minimum discount factor *is* monotonic if the number of firms is at least five. See Majerus (1988) and Martin (1989).

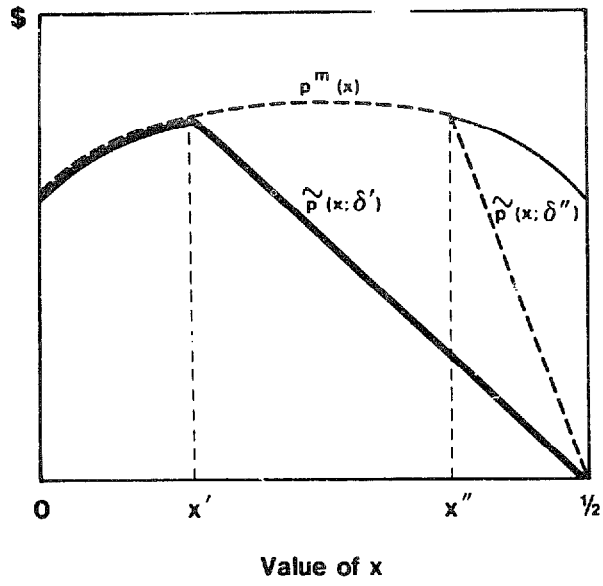

Fig. 3. Optimal collusive price for $\delta=\delta'$ and $\delta=\delta''$ where $\delta'<\delta''<\delta(1/2)$.

$$\tilde{p}(x,\delta)=\begin{cases}p^m(x) & \forall x\in[0,x^0],\\ b(1-2x)\mu(\delta) & \forall x\in(x^0,1/2).\end{cases}$$

Differentiating $\tilde{p}(x;\delta)$ with respect to x for all $x\in(x^0,1/2)$ gives us $\partial\tilde{p}(x;\delta)/\partial x<0$ for all $x\in(x^0,1/2)$. The best collusive price declines monotonically as the products become better substitutes.[9] When the products are strong substitutes, the incentive to deviate from the collusive agreement tends to outweigh the incentive to collude. Because this gain from defection must be reduced in order to sustain collusion, firms must lower price relative to the situation when they are producing weaker substitutes.

Given Property 4, one can graph $\tilde{p}(x;\delta)$ as a function of the product location, x. Fig. 3 depicts the best collusive price for different levels of the discount factor. For $\delta>\hat{\delta}(1/2)$, firms are able to support $\tilde{p}(x;\delta)=p^m(x)\,\forall x\in[0,1/2)$. Recall that $\partial p^m(x)/\partial x>0\ \forall x\in[0,1/4)$ and $\partial p^m(x)/\partial x<0\ \forall x\in[1/4,1/2)$. Joint profits are thus maximized at $x=1/4$. For a sufficiently low discount factor such as δ' or δ'' where $\delta'<\delta''<\hat{\delta}(1/2)$, $\tilde{p}(x;\delta)$ consists of two segments. In the case of $\delta=\delta'$, we observe that $\tilde{p}(x;\delta)=p^m(x)\ \forall x\in[0,x']$ and $\tilde{p}(x;\delta)=b(1-2x)\mu(\delta)\ \forall x\in(x',1/2)$. While we observe a similar tendency for $\delta=b^n, p^m(x)$ is supported for a wider set of product locations, $[0,x^n]$, due to the fact that $\delta'<\delta^n$.

[9]Note that the monotonic relationship between price and location is true only for those locations which do not support the joint profit maximizing price, i.e., $x\in(x^0,1/2)$. As can be seen from fig. 3, the joint profit maximizing price, $p^m(x)$, is not necessarily monotonic in locations.

It is worthwhile to note that, for a sufficiently low discount factor, firms fail to support the joint profit maximizing price, $p^m(x)$, if their products are strong substitutes. This is because of the strong incentive to deviate from the collusive price for strongly substitutable products. In this case, the only way for firms to pursue a collusive outcome is to make deviation less attractive by setting the collusive price below $p^m(x)$.

7. Conclusion

This paper investigated the relationship between the degree of product differentiation and the ability of firms to collude. Extending the spatial competition model of Hotelling (1929), the industry is modelled as a non-cooperative repeated price game in which products are horizontally differentiated. In the setting where firms can support collusion using a 'trigger' strategy, we find that the unconstrained joint profit maximum is supported as an equilibrium outcome if firms sufficiently value future profits. The minimum discount factor that supports the joint profit maximum is found to be a strictly increasing function of product substitutability. This result is in contrast to that of Deneckere (1983) which found a non-monotonic relationship betwen the minimum discount factor and the product substitutability. Deneckere (1983) utilizes a multi-product demand function approach to represent product differentiation. Different assumptions on consumer behavior are thus responsible for the discrepancy in our results.

We further extended our analysis by examining the best collusive price that firms can achieve when the discount factor held by them is not high enough to support the joint profit maximum. Our findings suggest that in Hotelling's model of product differentiation, collusion is always more difficult to sustain as the degree of product differentiation becomes smaller.

One distinct advantage of Hotelling's model over the multi-product demand function model is that it allows product locations to be endogenized. Since product design is one of the major decisions that firms must make in differentiated products industries, endogenization of product location has significant implications in analyzing collusive firm behavior when consumers are endowed with heterogeneous tastes. Interesting issues concerning the optimal product choice of colluding firms and its implications for social welfare are analyzed in Chang (1989, 1990)

Appendix

Proof of Proposition 1. Define C_1 and C_2 as subsets of C such that $C_1 \equiv \{(x, 1-x) | x \in [0, 1/4)\}$ and $C_2 \equiv \{(x, 1-x) | x \in [1/4, 1/2)\}$, where $C_1 \cup C_2 = C$. We shall consider the joint profit maximizing price vector for each subset. Let us first make the following conjecture: *The entire market,* $[0, 1]$, *is served*

at the joint profit maximizing prices. We will later show that this conjecture is indeed correct.

Case 1. Consider all $(x, 1-x) \in C_1$. Given the above conjecture, the market is divided into two adjoining submarkets that just barely touch each other when joint profits are maximized. Each firm is thus a local monopolist. The market division is then defined by the marginal consumer, $\hat{z}$, who incurs the utility cost of k. If $\hat{z}$ purchases x_1, then his utility cost is $b(x-\hat{z})^2+p_1=k$. If he purchases x_2, then he incurs $b(1-x-\hat{z})^2+p_2=k$. Solving for the prices, p_1 and p_2, as functions of $\hat{z}$, we obtain

$$p_1 = k - b(x-\hat{z})^2, \tag{A.1}$$

$$p_2 = k - b(1-x-\hat{z})^2. \tag{A.2}$$

The joint profits of the firms when they charge p_1 and p_2 as defined in (A.1) and (A.2) are written as:

$$\hat{\pi}(\hat{z}) = p_1\hat{z} + p_2(1-\hat{z}) = [k-b(x-\hat{z})^2]\hat{z} + [k-b(1-x-\hat{z})^2](1-\hat{z}). \tag{A.3}$$

Taking the first order condition and solving for the optimal $\hat{z}$, we find that $\hat{z}^* = 1/2$ maximizes $\hat{\pi}(\hat{z})$. Since the products are located symmetrically, $\hat{z}^* = 1/2$ implies that the joint profit maximizing prices are symmetric as well: $p_1 = p_2 = p^m(x) = k - b[(1/2)-x]^2$ for all $(x, 1-x) \in C_1$.

We will now show for all $(x, 1-x) \in C_1$ that our initial conjecture of the entire market being served was indeed correct. If the prices were raised above $p^m(x)$, then the market would be separated into two submarkets with a set of consumers in between not served. When the firms raise their prices sufficiently so that the submarkets are just beginning to separate, we can express their joint profit function as follows:

$$\hat{\pi}(p_1, p_2) = p_1[x + \{(k-p_1)/b\}^{1/2}] + p_2[x + \{(k-p_2)/b\}^{1/2}]. \tag{A.4}$$

From (A.4) we find that $\partial\hat{\pi}(p^m(x), p^m(x))/\partial p_i < 0 \ \forall (x, 1-x) \in C_1$ and $i = 1, 2$. Thus, the firms have absolutely no incentive to raise the prices above $p^m(x)$, and the entire market is served at $(p^m(x), p^m(x))$.

Case 2. Consider all $(x, 1-x) \in C_2$. For all $(x, 1-x) \in C_2$, the joint profit function is strictly increasing in p_1 and p_2 until the entire market is just barely served. This implies that p_1 and p_2 should be raised until the utility cost of consumers on both edges of the market, (0, 1), reaches the reservation price, k. At this point, the prices are $p_1 = p_2 = p^m(x)$, where $p^m(x) = k - bx^2$.

Again, let us show for all $(x, 1-x) \in C_2$ that our initial conjecture

regarding the entire market being served was correct. Defining z_1 to be the leftmost consumer purchasing x_1, the demand for x_1 as firm 1 raises its price over $p^m(x)$ is $(\hat{z}-z_1)$, where $\hat{z}=(p_2-p_1)/[2b(1-2x)]+(1/2)$ and $z_1=x-[(k-p_1)/b]^{1/2}$. When z_2 is defined as the rightmost consumer purchasing x_2, the demand for x_2 as firm 2 raises its price over $p^m(x)$ is $(z_2-\hat{z})$, where $z_2=(1-x)+[(k-p_2)/b]^{1/2}$ and $\hat{z}$ is defined as before. Thus, z_1 and z_2 denote the locations of the edge consumers whose utility cost just reaches the reservation price level, k. The joint profit function, then, takes the following form:

$$\hat{\pi}(p_1,p_2)=(\hat{z}-z_1)p_1+(z_2-\hat{z})p_2. \tag{A.5}$$

Substituting expressions for z_1, z_2, and $\hat{z}$ into (A.5) and differentiating, we find that $\partial\hat{\pi}(p^m(x),p^m(x))/\partial p_i<0$ $\forall(x,1-x)\in C_2$ and $i=1,2$. There is no incentive for the firms to raise their prices above $p^m(x)$. Therefore, for $(x,1-x)\in C_2$, we find that $p_1=p_2=p^m(x)=k-bx^2$ maximize the joint profits, and the entire market is served at that price. Q.E.D.

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