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INTERTEMPORAL PRODUCT CHOICE AND ITS EFFECTS ON COLLUSIVE FIRM BEHAVIOR*

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This study analyzes the collusive behavior of firms in markets characterized by horizontal product differentiation. The special features are: 1) Product choice is endogenized, and 2) firms may redesign their products intertemporally at a fixed cost. The main objective is to examine the relationships between the extent of product differentiation, the product redesigning cost and the degree of collusion supportable. The findings indicate that flexible product design makes collusion more difficult to sustain since, in the event collusion breaks down, firms have an incentive to mitigate the severity of punishment by redesigning their products and thus reducing price competition.

1. INTRODUCTION

It is well known in the field of industrial organization that when a market is characterized by horizontal product differentiation, the pricing decision of a firm depends crucially on the substitutability of its rival firms' products. Of course, this relationship depends on the type of pricing behavior (conduct) firms pursue. The traditional approach is to examine this issue under the assumption that firms behave noncollusively. However, within the growing literature of the noncooperative theory of collusion, several researchers have recently examined the effect of the degree of product differentiation on the ability of firms to collude. The first attempt in this line of research was made by Deneckere (1983). Using the representative consumer model of product differentiation in the repeated games framework, Deneckere (1983) derived a trigger strategy equilibrium for both a price game and an output game when firms are exogenously given differentiated products. Chang (1991) approached the same problem using Hotelling's (1929) spatial competition model. (Similar approach has been used by Ross (1992).) Finally, Majerus (1988) and Martin (1989) extended the duopoly analysis of Deneckere (1983) to the more general oligopoly setting.

By assuming that each firm's product is exogenously determined and fixed over time, the above models do not address the issue of how changes in the degree of product differentiation come about. In actuality, product choice is not only endogenous but can be changed over time. If we can define a product characteristics space over which consumers with heterogeneous tastes are distributed,

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choosing a product design or changing an existing product's design is equivalent to locating or relocating a product in this product space. Assuming exogenous product differentiation amounts to preventing firms from redesigning their product; that is, relocating their products over time in product space. The ability of firms to relocate their product may be important in determining their ability to collude as, in the event collusion breaks down, firms can mitigate the severity of punishment by relocating their products so as to reduce price competition. By reducing the severity of punishment, firms can alter the degree of collusion which is sustainable.

This paper sets forth a supergame in which firms make price and product location decisions where relocation of one's product incurs a fixed cost f. One may infer that f measures the ease with which firms may redesign their products. Previous models with exogenous product differentiation can be interpreted as the case where f is sufficiently large so that relocation is never optimal.[2] Note that the level of f is dependent upon the existing technology. For instance, the wide spread use of CAD-CAM (Computer Aided Design and Manufacturing) may be responsible for a substantial reduction in the fixed cost of redesigning products: Altering a product design can be accomplished with relative ease nowadays.

My objective in this paper is two-fold. First, to assess how the initial degree of product differentiation affects the ability of firms to collude when they are capable of relocating in the event collusion breaks down. Second, to explore how the cost of relocating affects the ability of firms to collude. I pursue this objective in a sequence of steps. First, I construct an equilibrium in product choice and price when firms behave noncollusively. The second step is to prove the existence of an equilibrium which supports a collusive outcome. Finally, by examining the set of discount factors supporting the joint profit maximum as a subgame perfect equilibrium, I provide insight into the above issues.

The paper is organized as follows. In Section 2, I describe the model of differentiated products. Section 3 gives a formal description of the duopoly supergame. Preliminary results regarding the static game are discussed in Section 4. In Section 5, I characterize an equilibrium of the supergame in which firms compete in product choice and price intertemporally. Section 6 proves the existence of an equilibrium that supports collusion through the trigger strategies. Section 7 characterizes the punishment path that depends on the initial degree of product differentiation arising from collusion. Section 8 investigates the impact of the initial degree of product differentiation and fixed cost of relocation on the ability of firms to collude.

2. THE MODEL OF DIFFERENTIATED PRODUCTS

The differentiated commodity is represented in the product space X which is the unit interval [0, 1]. There are two firms, each producing only one product at constant (zero) marginal cost. The location of firm i is denoted by $x_i \in X$, where firms are numbered so that $x_1 \leq x_2$. Let Ω be the set of all possible pairs of product

[2] There is also a literature of spatial models in which locations, once chosen, can not be changed. See, for example, Lane (1980, Prescott and Visscher (1977), and Neven (1987).

locations: $\Omega \equiv \{(x_1, x_2) \mid x_1 \in [0, 1], x_2 \in [0, 1], x_1 \leq x_2\}$. We may also define a set C as a subset of Ω such that it contains only symmetric product location pairs, i.e., $C \equiv \{(x, 1 - x) \mid x \in [0, 1/2)\}$.

Consumers are uniformly distributed over [0, 1] where the location of a consumer represents his most preferred product. If consumer x^* purchases product $\hat{x}$ at price $p_{\hat{x}}$, his total cost is $p_{\hat{x}} + f(\hat{x}, x^*)$ where $f(\hat{x}, x^*)$ is the utility cost (in dollars) of purchasing a product different from the most preferred variety of x^*. Following the formulation of Neven (1985), I assume the function to be quadratic, $f(\hat{x}, x^*) = b(\hat{x} - x^*)^2$. Each consumer has a finite reservation price, k, so that she will buy one (zero) unit of the differentiated product if the delivered price is less than or equal to (greater than) k. I specify k to be finite but sufficiently high that the entire market is served. In particular, it is assumed throughout the paper that $(5/4)b \leq k < \infty$.[3] Finiteness of k is necessary to bound the strategy sets of the firms.

3. STRUCTURE OF THE GAME

Firms are given the initial locations, (x_1^0, x_2^0), as the initial condition for the infinite horizon game. In each period, firms play a two-stage game. In stage 1, firms simultaneously choose where to locate (or relocate), where x_i^t denotes the location of firm i in period t. Given a location pair, the firms then simultaneously choose price, p_i^t, in stage 2, where $p_i^t \in [0, k]$.[4] If $x_i^t \neq x_i^{t-1}$, then firm i incurs a fixed cost of relocating, f, in period t.

A location strategy for firm i is an infinite sequence of action functions, $\{S_i^t\}_{t=1}^{\infty}$, where $S_i^t : [0, 1]^2 \times [[0, 1]^2 \times [0, k]^2]^{t-1} \to [0, 1]$. A price strategy for firm i is an infinite sequence of action functions, $\{G_i^t\}_{t=1}^{\infty}$, where $G_i^t : [0, 1]^2 \times [[0, 1]^2 \times [0, k]^2]^{t-1} \times [0, 1]^2 \to [0, k]$. As thus defined, strategy sets include closed-loop strategies, where the state variables at t are (x_1^{t-1}, x_2^{t-1}). We restrict firms to using pure strategies.

Once firms make the location-price decisions, we may denote by $\hat{z}$ the consumer who is indifferent between purchasing x_1 and x_2 given prices (p_1, p_2) such that $p_1 + b(\hat{z} - x_1)^2 = p_2 + b(\hat{z} - x_2)^2$. Demand for x_1 is then defined by $\bar{z}(p_1, p_2; x_1, x_2)$ as follows:

$$\bar{z}(p_1, p_2; x_1, x_2) = \begin{cases} 0 & \text{if } \hat{z} \leq 0, \\ \hat{z} & \text{if } 0 < \hat{z} < 1, \\ 1 & \text{if } \hat{z} \geq 1. \end{cases} \tag{1}$$

Demand for x_2 is then simply $[1 - \bar{z}(p_1, p_2; x_1, x_2)]$. Letting x_i^t and p_i^t be the realization of S_i^t and G_i^t in period t, we may define the period t profits of firm i as follows:

[3] $k \geq (5/4)b$ is necessary for the entire market, [0, 1], to be served in equilibrium when firms do not collude. See Chang (1991) for details.

[4] While product selection is flexible in that it can be changed every period, two-stage modelling allows us to capture the idea that price is more flexible than product location: Prices can be changed in stage 2 given fixed product locations.

(2) $$\pi_1(x_1^t, x_2^t, p_1^t, p_2^t) = p_1^t \bar{z}(x_1^t, x_2^t, p_1^t, p_2^t) - \alpha f,$$

(3) $$\pi_2(x_1^t, x_2^t, p_1^t, p_2^t) = p_2^t[1 - \bar{z}(x_1^t, x_2^t, p_1^t, p_2^t)] - \alpha f,$$
$$t = 1, \dots, \infty, \qquad i = 1, 2,$$

where $\alpha = 0$ if $x_i^t = x_i^{t-1}$, and $\alpha = 1$ if $x_i^t \neq x_i^{t-1}$. Firm i incurs a fixed cost of relocating, f, if his new location in period t differs from the previous location. Since the profits in period t are dependent on (x_1^{t-1}, x_2^{t-1}) through α, the structural time-dependency is inherent. Firms have a common discount factor, δ, and adopt a strategy vector $\sigma = (\sigma_1, \sigma_2)$, where $\sigma_i = \{S_i^1, G_i^1, S_i^2, G_i^2, \dots, S_i^t, G_i^t, \dots\}$ is the strategy of firm i. In period τ, firm i maximizes the present value of its future profits starting from that period:

$$\sum_{t=\tau}^{\infty} \delta^{t-\tau} \pi_i(\sigma^t),$$

where $\sigma^t = (\sigma_1^t, \sigma_2^t) = (S_1^t, G_1^t, S_2^t, G_2^t)$. The equilibrium concept is that of closed-loop subgame perfect equilibrium. Closed-loop subgame perfect equilibrium in our model is a pair of decision rules specifying the locations and prices that form mutual best responses in every period for every possible state.

4. ONE-SHOT PRICE GAME

The purpose of this section is to introduce certain payoff functions which will be useful later on. The one-shot price game in my model is identical to stage 2 of Neven (1985) in that firms simultaneously choose prices given fixed locations. Neven (1985) showed that the Nash equilibrium in the price game entails a pair of prices $(\bar{p}(x_1, x_2), \bar{p}_2(x_1, x_2))$ such that $\bar{p}_1(x_1, x_2) = (2b/3)(x_2 - x_1) + (b/3)(x_2^2 - x_1^2)$ and $\bar{p}_2(x_1, x_2) = (4b/3)(x_2 - x_1) - (b/3)(x_2^2 - x_1^2)$ for $(x_1, x_2) \in \Omega$. The Nash equilibrium payoffs for stage 2 are defined as follows:

(4) $$v_1^N(x_1, x_2) = \pi_1(\bar{p}_1, \bar{p}_2; x_1, x_2) = \bar{p}_1(x_1, x_2)\bar{z}(x_1, x_2),$$

(5) $$v_2^N(x_1, x_2) = \pi_2(\bar{p}_1, \bar{p}_2; x_1, x_2) = \bar{p}_2(x_1, x_2)[1 - \bar{z}(x_1, x_2)].$$

Let $\psi_i(p_j)$ represent the one-period best response price of firm i for any given price of firm j, $p_j \in [0, k]$ such that $\pi_i(\psi_i(p_j), p_j; x_i, x_j) = \max_{p_i} \pi_i(p_i, p_j; x_i, x_j)$. $v_i^d(p_j; x)$ denotes the best response payoff to firm i when products are symmetrically located and firm j charges p_j: $v_i^d(p_j; x) \equiv \pi_i(\psi_i(p_j), p_j; x, 1 - x)$.

Denote by $\hat{\pi}(p_1, p_2)$ the joint profits of the cartel: $\hat{\pi}(p_1, p_2) = \pi_1(p_1, p_2) + \pi_2(p_1, p_2)$. The next proposition characterizes a pair of prices that maximizes the joint profits for symmetrically located firms.

PROPOSITION 1. (Chang 1991). *For all* $(x_1, x_2) \in C$ *and* $k \geq (5/4)b$, *there exists a price* $p^m(x)$ *such that*

$$\hat{\pi}(p^m(x), p^m(x)) = \max_{p_1, p_2} \hat{\pi}(p_1, p_2), \text{ where } p^m(\hat{x}) = \begin{cases} k - b[(1/2) - \hat{x}]^2 & \text{for all } \hat{x} \in [0, 1/4) \\ k - b\hat{x}^2 & \text{for all } \hat{x} \in [1/4, 1/2). \end{cases}$$

5. FEEDBACK EQUILIBRIUM

In this section, I characterize closed-loop subgame perfect equilibrium of the supergame for when firms do not collude. A careful construction and characterization of the noncollusive equilibrium is important to my analysis due to the nature of the collusive strategies adopted by the firms. The collusive strategies firms use in my model are the Friedman (1971)-type grim trigger strategies, in which firms permanently revert to a simple repetition of static equilibrium as soon as cheating is detected. In our case, this punishment path corresponds to the noncollusive equilibrium defined over both product design and price. Prior to proving the existence of collusion and examining its sustainability, it is thus essential that we carefully construct and characterize the noncollusive equilibrium in this intertemporal environment.

Given the initial pair of locations, (x_1^0, x_2^0), firms compete each period t (≥ 1) in both product location and price throughout the entire horizon. The strategies of the firms are feedback strategies in which the location choice of a firm in period t is dependent only upon the current locations of the firms (x_1^{t-1}, x_2^{t-1}). In other words, a firm's choice of whether or not to alter its product design in any period depends solely on current designs of the existing products. Once product choices are made, prices are then determined in stage 2 as functions of the new locations (x_1^t, x_2^t). The relevant solution concept is closed-loop subgame perfect equilibrium. From now on, I shall refer to closed-loop subgame perfect equilibrium in feedback strategies as the feedback equilibrium.

The feedback equilibrium in my model consists of a pair of decision rules that specifies for each firm what its new location should be, and a pair of prices, $p_1(x_1^t, x_2^t)$ and $p_2(x_1^t, x_2^t)$. It is shown in Theorem 1 that the Nash equilibrium prices, $\bar{p}_1(x_1^t, x_2^t)$ and $\bar{p}_2(x_1^t, x_2^t)$, are, in fact, a part of the feedback equilibrium. The relevant payoff functions of the firms located at (x_1^t, x_2^t) and charging $(\bar{p}_1, \bar{p}_2)$ are then $v_1^N(x_1^t, x_2^t)$ and $v_2^N(x_1^t, x_2^t)$ as defined in (4) and (5).

Prior to deriving the feedback equilibrium for the supergame, let me briefly discuss the optimal location choice of a firm in the absence of the relocation cost. Given $x_j \in [0, 1]$, I denote by $x_i^*(x_j)$ the optimal location of firm i when Nash equilibrium pricing is used in stage 2 such that $v_i^N(x_i^*(x_j), x_j) \geq v_i^N(x_i, x_j)$ $\forall$ $x_i \in [0, 1]$, $i, j = 1, 2$, $i \neq j$. Since a firm's Nash equilibrium payoff is strictly increasing in its distance from the other firm, it is easy to see that $x_i^*(x_j) = 0$ for $x_j \in (1/2, 1]$, and $x_i^*(x_j) = 1$ for $x_j \in [0, 1/2]$. This implies that if a firm ever finds it optimal to relocate, there is a unique location to which it will move—the end of the market that maximizes its distance from its opponent. The intuition behind this result is straightforward. Given that Nash equilibrium pricing prevails in the second stage, price competition in that stage will be more severe as products are closer substitutes. Consequently, a firm's optimal relocation choice is to maximally differentiate its product from the rival product so as to minimize the severity of price competition in stage 2.[5]

Before presenting Theorem 1, I define the following terms for notational convenience: $\bar{v}_i(x_i, x_j) \equiv v_i^N(x_i^*(x_j), x_j) - v_i^N(x_i, x_j)$, and $\underline{v}_i(x_i) \equiv v_i^N(0, 1) - v_i^N(x_i, x_j^*(x_i))$, $i, j = 1, 2$, $i \neq j$. Given firm j remains at x_j, $\bar{v}_i(x_i, x_j)$ is the single period gain in firm i's profits from relocating. Likewise, if firm j relocates to x_j^*, the relevant gain in firm i's profit from relocating is $\underline{v}_i(x_i)$. It is straightforward to show that $\underline{v}_i = \max\{\underline{v}_1, \underline{v}_2\} \leq \max\{\bar{v}_1, \bar{v}_2\} = \bar{v}_i$ for all $(x_1, x_2) \in \Omega$ and $i = 1$ or 2:[6] The firm which has $\max\{\underline{v}_1, \underline{v}_2\}$ is also the firm with $\max\{\bar{v}_1, \bar{v}_2\}$. Furthermore, such a firm always gains more from relocating when the rival remains at its current location than when it relocates.

THEOREM 1. *$(\phi_1^i(x_1^{t-1}, x_2^{t-1}), p_1(x_1^t, x_2^t); \phi_2^i(x_1^{t-1}, x_2^{t-1}), p_2(x_1^t, x_2^t))$ form a feedback equilibrium, where*

$$(6)\qquad (\phi_1^i(\cdot), \phi_2^i(\cdot)) = \begin{cases} (0, 1) & \forall (x_1^{t-1}, x_2^{t-1}) \in \lambda_1, \\ (x_1^*, x_2^{t-1}) & \forall (x_1^{t-1}, x_2^{t-1}) \in \lambda_2 \cup \lambda_4^2, \\ (x_i^*, x_j^{t-1}) & \forall (x_1^{t-1}, x_2^{t-1}) \in \lambda_4^4, \\ (x_1^{t-1}, x_2^*) & \forall (x_1^{t-1}, x_2^{t-1}) \in \lambda_3 \cup \lambda_4^3, \\ (x_1^{t-1}, x_2^{t-1}) & \forall (x_1^{t-1}, x_2^{t-1}) \in \lambda_4^1, \end{cases}$$

and

$$(7)\qquad p_1(x_1^t, x_2^t) = \bar{p}_1(x_1^t, x_2^t) = (2b/3)(x_2^t - x_1^t) + (b/3)[(x_2^t)^2 - (x_1^t)^2],$$

$$(8)\qquad p_2(x_1^t, x_2^t) = \bar{p}_2(x_1^t, x_2^t) = (4b/3)(x_2^t - x_1^t) - (b/3)[(x_2^t)^2 - (x_1^t)^2],$$

$$i, j = 1, 2,\ i \neq j, \textit{ and } t = 1, 2, \ldots, \infty,$$

where

$$\lambda_1 \equiv \{(x_1, x_2) \in \Omega \mid [1/(1-\delta)]\underline{v}_1(x_1) > f \textit{ and } [1/(1-\delta)]\underline{v}_2(x_2) > f\}$$

$$\lambda_2 \equiv \{(x_1, x_2) \in \Omega \mid [1/(1-\delta)]\underline{v}_1(x_1) > f \textit{ and } [1/(1-\delta)]\underline{v}_2(x_2) \leq f\}$$

$$\lambda_3 \equiv \{(x_1, x_2) \in \Omega \mid [1/(1-\delta)]\underline{v}_1(x_1) \leq f \textit{ and } [1/(1-\delta)]\underline{v}_2(x_2) > f\}$$

$$\lambda_4 \equiv \{(x_1, x_2) \in \Omega \mid [1/(1-\delta)]\underline{v}_1(x_1) \leq f \textit{ and } [1/(1-\delta)]\underline{v}_2(x_2) \leq f\},$$

and

[5] The notion of maximal differentiation is clear when the market is the unit interval (as in my case) because of the endpoint effect. The notion becomes somewhat less clearcut once one assumes a circular market. In a circular market, firms (duopolists) face competition from both directions unlike in the linear market where competition comes from only one direction. Since there are no endpoints in the circular market, maximal differentiation in a duopoly setting would entail two firms locating exactly opposite from one another on the circle.

[6] The proof of this result, which is straightforward but tedius, is available from the author on request.

$$\lambda_4^1 \equiv \{(x_1, x_2) \in \lambda_4 \mid [1/(1-\delta)]\bar{v}_1(x_1, x_2) \le f \text{ and } [1/(1-\delta)]\bar{v}_2(x_1, x_2) \le f\}$$

$$\lambda_4^2 \equiv \{(x_1, x_2) \in \lambda_4 \mid [1/(1-\delta)]\bar{v}_1(x_1, x_2) > f \text{ and } [1/(1-\delta)]\bar{v}_2(x_1, x_2) \le f\}$$

$$\lambda_4^3 \equiv \{(x_1, x_2) \in \lambda_4 \mid [1/(1-\delta)]\bar{v}_1(x_1, x_2) \le f \text{ and } [1/(1-\delta)]\bar{v}_2(x_1, x_2) > f\}$$

$$\lambda_4^4 \equiv \{(x_1, x_2) \in \lambda_4 \mid [1/(1-\delta)]\bar{v}_1(x_1, x_2) > f \text{ and } [1/(1-\delta)]\bar{v}_2(x_1, x_2) > f\}.$$

PROOF. Available upon request.

Since firms compete in price given a pair of locations (x_1^t, x_2^t) in any given period t, it is trivial to support Nash equilibrium prices as a part of any feedback equilibrium. When Nash equilibrium is established in the price game in stage 2, the payoff functions facing the firms in stage 1 are $v_1^N(x_1^t, x_2^t)$ and $v_2^N(x_1^t, x_2^t)$. Given the current locations (x_1^{t-1}, x_2^{t-1}), firm i $(i = 1, 2)$ has two alternatives: It may remain at x_i^{t-1} or it may relocate to x_i^*. In making this decision, firm i takes firm j's location choice as given and compares the benefit and cost of redesigning its product. The benefit is that redesigning one's product reduces the severity of future price competition thereby improving its stream of discounted profits. On the cost side, it must incur the one-time fixed cost of relocation, f. Thus, firm i optimally relocates in a given period if and only if the discounted sum of all future gains from relocating this period exceeds the cost of relocation. Since both firms face two location possibilities, in any period there are only four candidates for the new product location pair that could emerge from the current locations (x_1^{t-1}, x_2^{t-1}): (x_1^{t-1}, x_2^{t-1}), (x_1^{t-1}, x_2^*), (x_1^*, x_2^{t-1}), and (0, 1). The feedback equilibrium locations constructed in Theorem 1 then form mutual best responses to each other given current locations and the size of relocation cost, f. Furthermore, these locations are stationary in that the firms relocate at most once in equilibrium. In other words, if it is ever optimal to relocate, a firm moves at once to the optimal location and remains there for the rest of the horizon.

When f is very small that $(x_1^{t-1}, x_2^{t-1}) \in \lambda_1$, both firms optimally redesign their products away from each other, because the reduction in the severity of future price competition more than compensates for the one-time relocation cost: the equilibrium location pair is (0, 1). For a moderate value of f, we may have $(x_1^{t-1}, x_2^{t-1}) \in \lambda_2$ or $(x_1^{t-1}, x_2^{t-1}) \in \lambda_3$ depending on the current locations. In either case, the incentives to redesign differ among the firms. One firm may find it worthwhile to redesign its product for the benefit to him from reduced price competition exceeds the cost of its relocation, while the other firm prefers to remain at its current location: the equilibrium location pair is (x_1^*, x_2^{t-1}) if $(x_1^{t-1}, x_2^{t-1}) \in \lambda_2$ and (x_1^{t-1}, x_2^*) if $(x_1^{t-1}, x_2^{t-1}) \in \lambda_3$.

When f is sufficiently large that $(x_1^{t-1}, x_2^{t-1}) \in \lambda_4$, neither firm wishes to relocate if the rival firm is relocating. However, if a firm believes that its rival will not relocate, it may optimally relocate to reduce price competition. This set of sufficiently large f's can then be further divided in examining this incentive. Firstly, if f is still relatively small within this set that $(x_1^{t-1}, x_2^{t-1}) \in \lambda_4^4$, each firm finds it beneficial to relocate unilaterally given that the other firm remains at its current location. Thus, both (x_1^*, x_2^{t-1}) and (x_1^{t-1}, x_2^*) can be supported as feedback equilibria. Secondly, if f is relatively moderate that $(x_1^{t-1}, x_2^{t-1}) \in \lambda_4^2$ or $(x_1^{t-1}, x_2^{t-1}) \in \lambda_4^3$, unilateral relocation is optimal for only one of the firms depending on their current locations. Finally, when f is extremely large that $(x_1^{t-1}, x_2^{t-1}) \in \lambda_4^1$, the potential gains from reduced price competition is not worth the one-time cost of relocation f. In this case, both firms prefer to maintain current product designs, (x_1^{t-1}, x_2^{t-1}).

One thing to note from Theorem 1 is that there exist multiple equilibria for all $(x_1^{t-1}, x_2^{t-1}) \in \lambda_4^4$. While both (x_1^*, x_2^{t-1}) and (x_1^{t-1}, x_2^*) can be supported as an equilibrium, the feedback equilibrium was specified to be such that $(\phi_1^i(x_1^{t-1}, x_2^{t-1}), \phi_2^i(x_1^{t-1}, x_2^{t-1})) = (x_i^*, x_j^{t-1})$ for all $(x_1^{t-1}, x_2^{t-1}) \in \lambda_4^4$. Thus, the superscript i of $(\phi_1^i(\cdot), \phi_2^i(\cdot))$ denotes the firm that does the relocating at cost f when the products are currently located in λ_4^4. This specification becomes useful when the feedback equilibrium defined in Theorem 1 is used for the punishment path that supports collusion.

6. INTERTEMPORAL PRODUCT CHOICE AND COLLUSION

Using the feedback equilibrium as a punishment phase, I shall construct an equilibrium in which firms collude over both product choice and price. In the beginning of the game, we assume that firms propose a pair of collusive locations, $\hat{x}_1$ and $\hat{x}_2$. Since both firms are identical in all respects, it is natural to consider a pair of symmetric locations: $\hat{x}_1 = \hat{x}, \hat{x}_2 = 1 - \hat{x}$. The initial cost of introducing the products is assumed to be sunk. However, once the products are introduced, relocating the existing product incurs a fixed cost, f. Given the proposed locations, $(\hat{x}, 1 - \hat{x})$, firms try to support some common collusive price, $\hat{p} \in (\bar{p}(\hat{x}), p^m(\hat{x})]$. If the firms adhere to the collusive agreement, each firm earns the collusive payoff, $\hat{v}_k(\hat{p}; \hat{x})$, in every period that they collude, where $\hat{v}_k(\hat{p}; \hat{x}) \equiv \pi_k(\hat{p}; \hat{x}, 1 - \hat{x}) = (1/2)\hat{p}$, $k = 1, 2$. It is important to note that in my model of collusion we need only consider those $\hat{x}$ and $\hat{p}$ that satisfy $\hat{v}_k(\hat{p}; \hat{x}) > v_k^N(0, 1)$, $k = 1, 2$. Intuitively, a firm would not consider colluding in location and price unless the proposed collusive agreement $(\hat{x}, \hat{p})$ satisfies $\hat{v}_k(\hat{p}; \hat{x}) > v_k^N(0, 1)$. Otherwise, firms would prefer to locate themselves at the extreme ends of the market and compete in price. This would be a repetition of Neven's (1985) static subgame perfect equilibrium in the infinite horizon setting and would certainly form a subgame perfect equilibrium in the supergame by Theorem 1. Since the objective is to analyze the collusive behavior of the firms, we restrict our attention to the set of $\hat{x}$ and $\hat{p}$ that satisfies $\hat{v}_k(\hat{p}; \hat{x}) > v_k^N(0, 1)$.[7]

Let us now consider the following class of strategies for firms 1 and 2, respectively:

$$S_1^1 = \hat{x}; \tag{9}$$

$$G_1^1 = \begin{cases} \hat{p} & \text{if } (x_1^1, x_2^1) = (\hat{x}, 1 - \hat{x}); \\ \bar{p}_1(x_1^1, x_2^1) & \text{otherwise;} \end{cases}$$

[7] To see that the set of $\hat{x}$'s and $\hat{p}$'s that satisfy the restriction is not empty, consider the joint profit maximizing price, $p^m(\hat{x})$. It can be easily seen that $\hat{v}_k(p^m(\hat{x}), \hat{x}) > v_k^N(0, 1)$ for all $\hat{x} = \in [0, 1/2)$.

$$S_1^t = \begin{cases} \hat{x} & \text{if } (x_1^\tau, x_2^\tau) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^\tau, p_2^\tau) = (\hat{p}, \hat{p}),\ \tau = 1, 2, \dots, t-1; \\ \phi_1^1(x_1^{t-1}, x_2^{t-1}) & \text{if there exists some } s,\ 1 \le s \le t-1, \text{ such that } (x_1^\tau, x_2^\tau) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^\tau, p_2^\tau) = (\hat{p}, \hat{p}) \text{ for } \tau = 1, 2, \dots, s-1, \text{ and either } (x_1^s \ne \hat{x}, x_2^s = 1-\hat{x}) \text{ or } (x_1^s, x_2^s) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^s \ne \hat{p}, p_2^s = \hat{p}); \\ \phi_1^2(x_1^{t-1}, x_2^{t-1}) & \text{otherwise;} \end{cases}$$

$$G_1^t = \begin{cases} \hat{p} & \text{if } (x_1^\tau, x_2^\tau) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^\tau, p_2^\tau) = (\hat{p}, \hat{p}) \text{ for } \tau = 1, 2, \dots, t-1, \text{ and } (x_1^t, x_2^t) = (\hat{x}, 1-\hat{x}); \\ \bar{p}_1(x_1^t, x_2^t) & \text{otherwise;} \quad t = 2, 3, \dots, \infty. \end{cases}$$

$$(10) \qquad S_2^1 = 1 - \hat{x};$$

$$G_2^1 = \begin{cases} \hat{p} & \text{if } (x_1^1, x_2^1) = (\hat{x}, 1-\hat{x}); \\ \bar{p}_2(x_1^1, x_2^1) & \text{otherwise;} \end{cases}$$

$$S_2^t = \begin{cases} 1-\hat{x} & \text{if } (x_1^\tau, x_2^\tau) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^\tau, p_2^\tau) = (\hat{p}, \hat{p}),\ \tau = 1, 2, \dots, t-1; \\ \phi_2^1(x_1^{t-1}, x_2^{t-1}) & \text{if there exists some } s,\ 1 \le s \le t-1, \text{ such that } (x_1^\tau, x_2^\tau) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^\tau, p_2^\tau) = (\hat{p}, \hat{p}) \text{ for } \tau = 1, 2, \dots, s-1, \text{ and either } (x_1^s \ne \hat{x}, x_2^s = 1-\hat{x}) \text{ or } (x_1^s, x_2^s) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^s \ne \hat{p}, p_2^s = \hat{p}); \\ \phi_2^2(x_1^{t-1}, x_2^{t-1}) & \text{otherwise;} \end{cases}$$

$$G_2^t = \begin{cases} \hat{p} & \text{if } (x_1^\tau, x_2^\tau) = (\hat{x}, 1-\hat{x}) \text{ and } (p_1^\tau, p_2^\tau) = (\hat{p}, \hat{p}) \text{ for } \tau = 1, 2, \dots, t-1, \text{ and } (x_1^t, x_2^t) = (\hat{x}, 1-\hat{x}); \\ \bar{p}_2(x_1^t, x_2^t) & \text{otherwise;} \quad t = 2, 3, \dots, \infty. \end{cases}$$

In period 1, firms introduce their products at the agreed-upon locations, $(\hat{x}, 1-\hat{x})$, in stage 1. If both firms colluded in location, in stage 2 they set price at the collusive level, $\hat{p}$. In stage 1 of period t ($t > 1$), firms remain at $(\hat{x}, 1-\hat{x})$ as long as no one has deviated in periods 1 through $t-1$. If firm 1 was the first to deviate in location or price, they immediately revert to the feedback equilibrium locations $(\phi_1^1(x_1^{t-1}, x_2^{t-1}), \phi_2^1(x_1^{t-1}, x_2^{t-1}))$. The superscript 1 implies that firm 1 was the defector and, thus, firm 1 should relocate on the punishment path if it ever becomes necessary. If either firm 2 unilaterally defects or both defect simultaneously, $(\phi_1^2(x_1^{t-1}, x_2^{t-1}), \phi_2^2(x_1^{t-1}, x_2^{t-1}))$ is the locational equilibrium chosen. In stage 2 of period t, firms charge $\hat{p}$ if no one deviated in the past and both firms colluded in location in stage 1 of that period. Otherwise, they revert to the Nash equilibrium prices based on the location decisions made in stage 1.

While deviation can occur either in location or price, it is important to note that there is no incentive to deviate in location when $(\hat{x}, \hat{p})$ satisfies $\hat{v}_i(\hat{p}; \hat{x}) > v_i^N(0, 1)$. If firm i ever deviates in location, the best it can do is to relocate to $x_i^*(\hat{x}_j)$ and incur f. It then earns either $v_i^N(x_i^*, x_j)$ or $v_i^N(0, 1)$ (if j relocates in response to i's relocation) for the remainder of the horizon. Since $\hat{v}_i(\hat{p}; \hat{x}) > v_i^N(0, 1) > v_i^N(x_i^*, x_j)$, it is obvious that the defection payoff is strictly dominated by the collusive payoff for all values of δ. In the remainder of the paper, we need only concern ourselves with the incentive to deviate via price.

Using the strategies specified in (9) and (10), the collusive outcome, $(\hat{x}, 1 - \hat{x})$ and $\hat{p}$, can be sustained as a closed-loop subgame perfect equilibrium outcome in the supergame if and only if

$$[1/(1 - \delta)]\hat{v}_i(\hat{p}; \hat{x}) \geq v_i^d(\hat{p}; \hat{x}) + \delta\Gamma_i(\hat{x}, 1 - \hat{x}). \tag{11}$$

$\hat{v}_i(\hat{p}; \hat{x})$ is the collusive payoff for firm i in each period if the firms collude over both location and price. $v_i^d(\hat{p}; \hat{x})$ is the single-period payoff to firm i if it optimally (and unilaterally) deviates in price in some period s. $\Gamma_i(\hat{x}, 1 - \hat{x})$ is the present value of firm i's stream of payoffs on the punishment path in periods $s + 1$ to ∞, discounted back to the beginning of period $s + 1$. This is the equilibrium payoff for the punishment path, where firms choose $(\phi_1^i(x_1^{t-1}, x_2^{t-1}), \bar{p}_1(x_1^t, x_2^t); \phi_2^i(x_1^{t-1}, x_2^{t-1}), \bar{p}_2(x_1^t, x_2^t))$ to achieve a feedback equilibrium for $t = s + 1, s + 2, \ldots, \infty$.

We are now ready to show the existence of an equilibrium in which firms collude over both product choice and price. (Proofs of Theorem 2 and Proposition 2 are provided in the Appendix.)

THEOREM 2. *There exist* $\Delta(\hat{x}, \hat{p}) \subset (0, 1)$ *such that* (9) *and* (10) *form a closed-loop subgame perfect equilibrium if and only if* $\delta \in \Delta(\hat{x}, \hat{p})$. *Furthermore, there exists* $\hat{\delta} \in (0, 1)$ *such that* $(\hat{\delta}, 1) \subset \Delta(\hat{x}, \hat{p})$.

Theorem 2 indicates that collusion over both product choice and price can be sustained in the long run if the firms sufficiently value future profits. While this result is a standard outcome in the literature of collusive supergames, the next result is somewhat surprising.

PROPOSITION 2. *There exist* $\underline{f}(\hat{x})$ *and* $\bar{f}(\hat{x})$ *such that* $\Delta(\hat{x}, \hat{p})$ *is not a connected subset of* $(0, 1)$ *for all* $f \in (\underline{f}(\bar{x}), \bar{f}(\hat{x}))$, *where* $\underline{f}(\hat{x}) < \bar{f}(\hat{x})$ *for all* $\hat{x} \in (0, 1/2)$.

It has been observed in repeated-game models of collusion that firms with a higher discount factor can support a collusive outcome that is at least as good as the outcome supported by a lower discount factor: The equilibrium payoff attainable is "monotonically increasing" with respect to the discount factor. This "monotonicity" result is quite intuitive in standard repeated games, since the increase in the discount factor implies that the firms value their future losses more heavily than the one-time gain from defection. As their valuation of future profits increases, a better collusive outcome is supported as an equilibrium. Contrary to this generally accepted result, the above finding suggests that under some circumstances, a lower discount factor supports a better equilibrium outcome than a higher discount factor:

The equilibrium payoff supportable is not necessarily monotonic in δ in our game. Since this result seems counter-intuitive at first glance, it is important that we understand what drives this phenomenon. In order for the trigger strategy to be a subgame perfect equilibrium, two conditions must be satisfied: Collusion must be profitable and punishment must be credible. For all $f \in (\underline{f}(\hat{x}), \bar{f}(\hat{x}))$, there exist a set of discount factors such that (0, 1) is the equilibrium punishment location pair, but collusion is not profitable with that location pair. In order for collusion to be profitable with that level of discount factor, punishment must involve relocation of the deviator only, i.e. $(0, 1 - \hat{x})$ or $(\hat{x}, 1)$. However, for this set of δ's the victimized firm finds it optimal to relocate on the punishment path given that the deviator relocates, i.e., (0, 1). Consequently, the victimized firm's relocation reduces the severity of price competition at no extra cost to the deviating firm. This reduces the cost to cheating and lowers the incentive to collude below the level that exists for a lower discount factor. Certainly for some lower discount factor, the victimized firm optimally maintains its initial location and collusion becomes profitable with $(0, 1 - \hat{x})$ or $(\hat{x}, 1)$ as a credible (feedback equilibrium) location pair. Therefore, a lower discount factor supports a better equilibrium outcome than a higher one. The nonmonotonic nature of the cartel-sustaining discount factor will become clearer once we explicitly characterize it for the joint profit maximizing price in Section 8.

7. CHARACTERIZATION OF THE PUNISHMENT PATH

When deviation occurs, the firms immediately revert to the punishment path which is characterized by the feedback equilibrium. Since the feedback strategies are dependent upon the current state of the game, the collusive location pair $(\hat{x}, 1 - \hat{x})$ determines the initial condition for the punishment path. It is thus straightforward that the initial choice of collusive locations has a direct impact on firms' locational behavior when collusion breaks down. Proposition 3 characterizes this relationship.

PROPOSITION 3. *There exists $\hat{f} > 0$, such that for all $f \in (0, \hat{f})$, there exist $\underline{x}(\delta; f)$ and $\bar{x}(\delta; f)$, where $0 < \underline{x}(\delta; f) < \bar{x}(\delta; f) < 1/2$ such that*

$$(12)\quad (\phi_1^i(\hat{x}), \phi_2^i(\hat{x})) = \begin{cases} (\hat{x},\ 1-\hat{x}) & \text{if } 0 \le \hat{x} \le \underline{x}(\delta; f), \\ \left.\begin{matrix} (0,\ 1-\hat{x}) & \text{for } i = 1 \\ (\hat{x},\ 1) & \text{for } i = 2 \end{matrix}\right\} & \text{if } \underline{x}(\delta; f) < \hat{x} < \bar{x}(\delta; f), \\ (0,\ 1) & \text{if } \bar{x}(\delta; f) \le \hat{x} < 1/2. \end{cases}$$

PROOF. Straightforward from Theorem 1 by taking $(x_1^{t-1}, x_2^{t-1}) = (\hat{x}, 1 - \hat{x})$.

Exogenously given a fixed cost of relocating, if the collusive arrangement entails the firms locating sufficiently far apart from one another (i.e. $\hat{x} \le \underline{x}(\delta; f)$), the punishment involves no relocation, $(\hat{x}, 1 - \hat{x})$. Since price competition is not very severe when firms are sufficiently far apart, the incremental gains from relocating to the optimal site do not justify the fixed cost of moving. Conversely, if the firms agree to locate sufficiently close to one another (that is, $\hat{x} \in [\bar{x}(\delta; f), 1/2)$), the punishment entails both firms relocating to (0, 1), thereby maximally differentiating their products. Recall that the Nash profits are close to zero when the firms locate their products right next to one another. Choosing $\hat{x}$ to be near 1/2 could potentially inflict severe damage on both firms if neither firm were to relocate in the event of a defection. However, for this very reason, firms optimally relocate to (0, 1) so as to reduce price competition. Finally, for $\hat{x} \in (\underline{x}(\delta; f), \bar{x}(\delta; f))$ such that the products are moderate substitutes, the feedback equilibrium has only the defecting firm relocating. Since both $(0, 1 - \hat{x})$ and $(\hat{x}, 1)$ can be enforced as an equilibrium for this interval, the existence of multiple equilibria presents us with a natural opportunity to impose an asymmetric punishment. To see the intuitive appeal of the asymmetric equilibrium on the punishment path, let firm 1 be the defector. The strategies specify $(\phi_1^1(\cdot), \phi_2^1(\cdot)) = (0, 1 - \hat{x})$ to be the locational equilibrium on the punishment path. Firm 1, the defector, relocates to 0 incurring the fixed cost of relocation, f, while firm 2 remains where it was prior to defection. From firm 2's perspective, the punishment imposes no relocation cost on itself. Furthermore, firm 1's relocation to 0 on the equilibrium path greatly reduces the degree of price competition which would be relatively severe if firm 1 remained at its existing location. Since only the defector bears the burden of relocating, it is quite obvious that the punishment is more severe on the defector.[8]

Figure 1 illustrates the relationship between $\underline{x}(\delta; f)$, $\bar{x}(\delta; f)$, and δ where $\underline{\delta}(x; f) = \underline{x}^{-1}(\delta; f)$ and $\bar{\delta}(x; f) = \bar{x}^{-1}(\delta; f)$. In Figure 1, area T_1 represents those $(\delta, \hat{x})$ pairs which support $(\hat{x}, 1 - \hat{x})$ as a credible (feedback equilibrium) punishment location, i.e., if collusion broke down, firms would not relocate. Area T_2 supports $(0, 1 - \hat{x})$ and area T_3 supports (0, 1). As one may expect, $\underline{x}(\delta; f)$ and $\bar{x}(\delta; f)$ decline in their values as δ increases. δ represents the trade-off between higher future profits resulting from relocation and the fixed cost of relocation, f. Since higher δ value implies that the firms find the benefits of relocation increasingly attractive, larger set of initial locations entail relocation as feedback equilibrium on the punishment path.

8. IMPACT OF PRODUCT LOCATION AND FIXED COST OF RELOCATION ON FIRMS' ABILITY TO COLLUDE

The analysis thus far has concentrated on constructing feedback equilibria and proving the existence of closed-loop subgame perfect equilibrium that supports collusion over product location and price. In this section, I shall examine the degree of collusion which can be sustained when firms may relocate their products intertemporally by incurring a fixed cost, f.

Recall from Section 6 that a closed-loop subgame perfect equilibrium is defined by $(\hat{x}, \hat{p})$ such that (9) and (10) form a subgame perfect equilibrium. Such an equilibrium was shown to exist for all $\delta \in \Delta(\hat{x}, \hat{p})$. The degree of collusion can then

[8] The idea that it is the defecting firm which relocates on the punishment path is somewhat reminiscent of the repentence strategy in Segerstrom (1988). Of course, the difference is that the eventual forgiving of the cheater incorporated in the repentence strategy is not available in my grim trigger strategy.

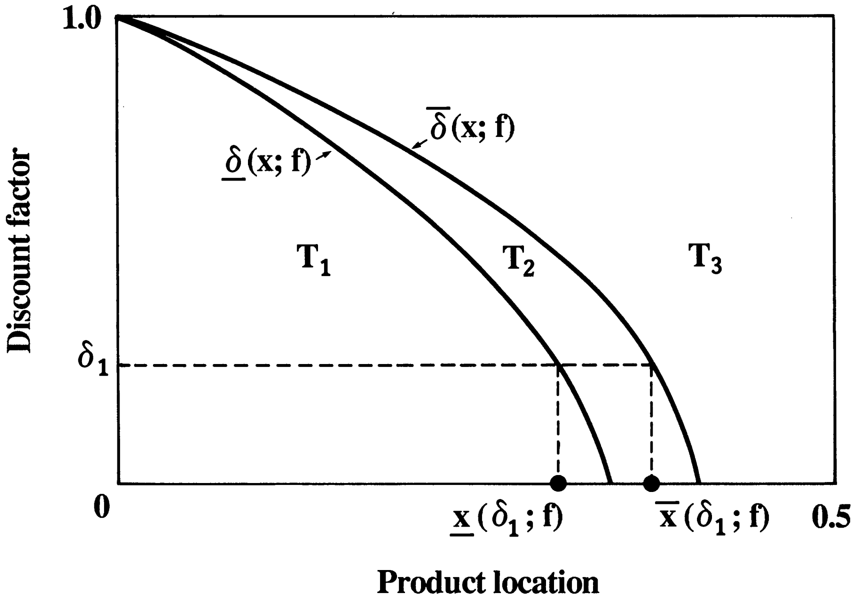

FIGURE 1

be measured by the size of the set $\Delta(\hat{x}, \hat{p})$. Two questions that I attempt to find answers for are: 1) How do initial product locations, $(\hat{x}, 1 - \hat{x})$, affect the ability of firms to collude? 2) How does the relocation cost, f, affect the firms' ability to collude? In order to investigate these issues, we specifically focus on the case where firms pursue a joint profit maximizing price as a part of collusive agreement, i.e., $\hat{p} = p^m(\hat{x})$. When firms pursue $\hat{p} = p^m(\hat{x})$ using the punishment path as defined in Section 7, it is straightforward that collusion is sustainable if and only if

$$(13) \quad [(\delta/1 - \delta)][\hat{v}_i(p^m(\hat{x})) - v_i^N(\hat{x}, 1 - \hat{x})] \geq v_i^d(p^m(\hat{x})) - \hat{v}_i(p^m(\hat{x})) \quad \text{for all} \quad \hat{x} \in [0, \underline{x}(\delta; f)),$$

$$(14) \quad [(\delta/1 - \delta)][\hat{v}_i(p^m(\hat{x})) - v_i^N(0, 1 - \hat{x})] + \delta f \geq v_i^d(p^m(\hat{x})) - \hat{v}_i(p^m(\hat{x})) \quad \text{for all} \quad \hat{x} \in [\underline{x}(\delta; f), \bar{x}(\delta; f)),$$

$$(15) \quad [(\delta/1 - \delta)][\hat{v}_i(p^m(\hat{x})) - v_i^N(0, 1)] + \delta f \geq v_i^d(p^m(\hat{x})) - \hat{v}_i(p^m(\hat{x})) \quad \text{for all} \quad \hat{x} \in [\bar{x}(\delta; f), 1/2).$$

These sustainability conditions simplify to $\delta \geq \delta_1(\hat{x})$ for $\hat{x} \in [0, \underline{x}(\delta; f))$, $\delta \geq \delta_2(\hat{x}; f)$ for $\hat{x} \in [\underline{x}(\delta; f), \bar{x}(\delta; f))$, and $\delta \geq \delta_3(\hat{x}; f)$ for $\hat{x} \in [\bar{x}(\delta; f), 1/2)$. $\delta_1(\hat{x})$ is independent of f and can be derived explicitly in terms of $\hat{x}$ (see Chang 1991). However, $\delta_2(\hat{x}; f)$ and $\delta_3(\hat{x}; f)$ are dependent upon the level of f and it is not possible to derive them as explicit functions of $\hat{x}$. Thus, instead of examining $\Delta(\hat{x}, \hat{p})$ analytically, we examine it graphically by simulating $\delta_2(\hat{x}; f)$ and $\delta_3(\hat{x}; f)$ via numerical analysis techniques for certain parameter values. In an attempt to

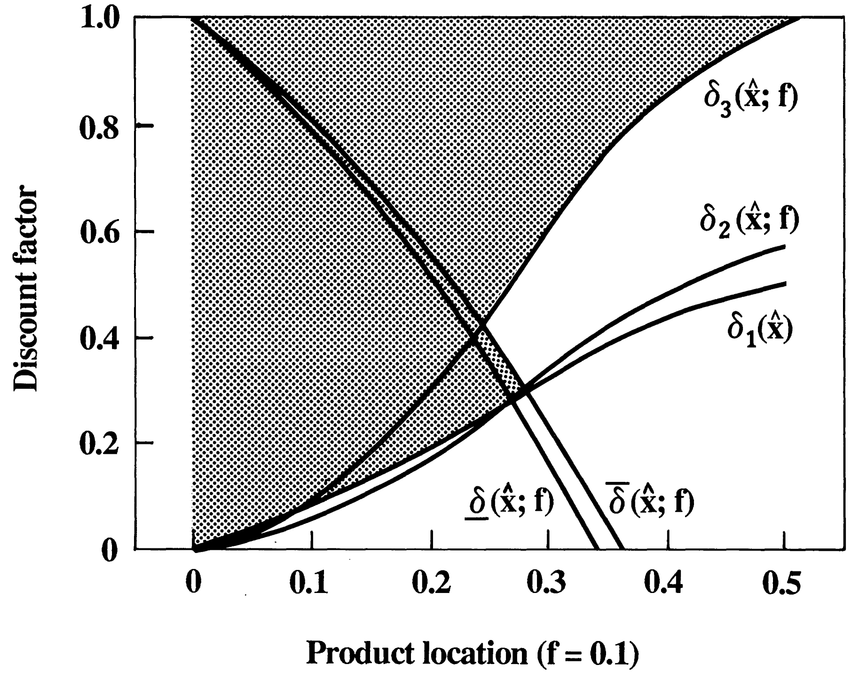

FIGURE 2

CARTEL-SUSTAINING DISCOUNT FACTORS FOR $f = 0.1$

examine $\Delta(\hat{x}, \hat{p})$ graphically, I assume $k = (5/4)b$ and $b = 1$. For $f = 0.1$ and $f = 0.3$, $\delta_1(\hat{x})$, $\delta_2(\hat{x}; f)$, and $\delta_3(\hat{x}; f)$ are drawn in Figures 2 and 3 along with $\underline{\delta}(\hat{x}; f)$ and $\bar{\delta}(\hat{x}; f)$ as defined in Section 7. The shaded regions are the set of $(\delta, \hat{x})$ that support collusion as subgame perfect equilibrium.

For those $(\delta, \hat{x})$ combinations which support $(\hat{x}, 1 - \hat{x})$ as credible punishment, i.e. area T_1, collusion is sustainable if and only if $\delta \geq \delta_1(\hat{x})$. For those $(\delta, \hat{x})$ combinations which support $(0, 1 - \hat{x})$ as credible punishment, i.e. area T_2, collusion is sustainable if and only if $\delta \geq \delta_2(\hat{x}; f)$. Likewise, for those $(\delta, \hat{x})$'s supporting (0, 1) as credible punishment, i.e., area T_3, collusion is sustainable if and only if $\delta \geq \delta_3(\hat{x}; f)$. Accordingly, the shaded areas in both figures represent the set of $(\delta, \hat{x})$'s which support collusion as a subgame perfect equilibrium. $\Delta(\hat{x}, p^m)$ is defined by the set of δ's that belong to the shaded area for a given *collusive* location pair, $(\hat{x}, 1 - \hat{x})$.

RESULT 1. *Collusion is more difficult to sustain for stronger substitutes.*

This result was verified in Chang (1991) for the case of exogenous product locations, i.e., f sufficiently large. For f sufficiently large, $(\hat{x}, 1 - \hat{x})$ is the only credible punishment for all $\hat{x} \in [0, 1/2)$ and for all δ's. In this case, $\delta \geq \delta_1(\hat{x})$ alone insures the sustainability of collusion. It is straightforward to show that $\delta_1(\hat{x})$ is monotonically increasing in $\hat{x}$ for $b > 0$ and $k \geq (5/4)b$ (Chang 1991). The intuitive

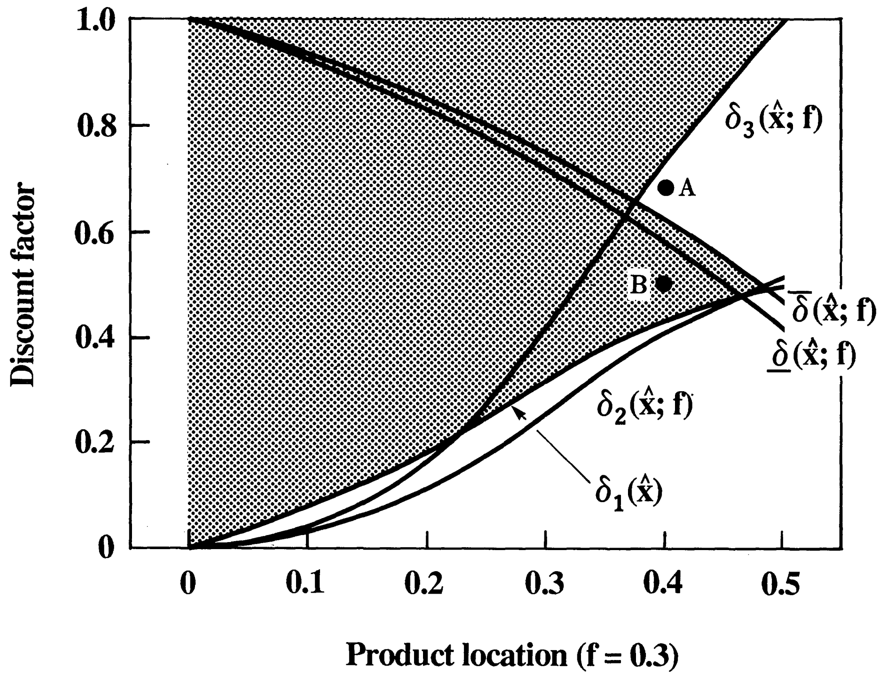

FIGURE 3

CARTEL-SUSTAINING DISCOUNT FACTORS FOR $f = 0.3$

reasoning behind this result is that as products become better substitutes, a defecting firm will take more of the market away from his rival. For when product designs are flexible (i.e., f is not large), collusion becomes even more difficult to sustain for stronger substitutes. This result can be confirmed by observing Figure 2 and Figure 3. A higher value of $\hat{x}$ implies that the products are closer substitutes. As $\hat{x}$ gets bigger, $\Delta(\hat{x}, p^m)$ declines in its size implying that collusion is more difficult to sustain. The intuition behind this result becomes clear as we relate it to Proposition 3. In Proposition 3, it was observed that firms producing strong substitutes redesign their products in the event of a defection, so as to reduce the severity of price competition on the punishment path. As the initial collusive products are better substitutes, the firms' incentive to redesign their products in the event of a defection increase. Since the degree of collusion sustainable is directly related to the severity of punishment, stronger substitutes find it more difficult to sustain collusion.

RESULT 2. *Collusion is more difficult to sustain with a lower fixed cost of relocation.*

In the case of a deviation, firms immediately revert to the punishment phase which entails a price war defined by Nash equilibrium pricing. Since Nash equilibrium prices are dependent upon the firms' locations and firms' profits are increasing in the distance between them, firms have two choices: Endure severe price competition by remaining at the initial location of $(\hat{x}, 1 - \hat{x})$, or relocate to the end of the market (i.e., $(0, 1 - \hat{x})$ or (0, 1)) at cost f and thereby reducing the severity of price competition. Then, for a given level of δ, a lower f implies that firms find it cheaper to relocate and avoid severe price competition—relocation becomes optimal for a larger set of initial products. Since this post-deviation relocation reduces the cost to cheating, collusion is more difficult to sustain as f decreases. This conclusion can be confirmed by examining the size of $\Delta(\hat{x}, p^m)$ in Figures, 2 and 3 for $f = 0.1$, and $f = 0.3$, respectively.

The intuition behind the "nonmonotonicity" of the cartel-sustaining discount factor (Proposition 2) can now be seen quite clearly. Consider a $(\delta, \hat{x})$ pair such as point A in Figure 3. Firms have a common discount factor of $\delta = 0.7$ and products are located at (0.4, 0.6) for $\hat{x} = 0.4$. Given these collusive locations, $p^m(\hat{x})$ can be supported as a collusive outcome if firms can ensure that either $(\hat{x}, 1 - \hat{x})$ or $(0, 1 - \hat{x})$ is to be used as punishment locations in case of a deviation—$\delta = 0.7$ is strictly greater than $\delta_1(\hat{x})$ and $\delta_2(\hat{x}; f)$ at $\hat{x} = 0.4$. However, since point A falls in area T_3, neither $(\hat{x}, 1 - \hat{x})$ nor $(0, 1 - \hat{x})$ is credible: At $\delta = 0.7$, firm 2 would optimally relocate to avoid the price competition on the punishment path. Thus, using (0, 1) as the only credible punishment location pair, $\delta = 0.7$ is not high enough to support $p^m(\hat{x})$ as a collusive outcome—$0.7 < \delta_3(\hat{x}; f)$. On the other hand, for a lower discount factor, $\delta = 0.5$, $p^m(\hat{x})$ is supportable as an equilibrium outcome. The reason is that at $\delta = 0.5$ and $\hat{x} = 0.4$, the credible punishment is $(\hat{x}, 1 - \hat{x})$. Since $0.5 > \delta_1(\hat{x})$, collusion can be supported. The "nonmonotonicity" result is due to the restriction imposed on the firms that they must satisfy the credibility of punishment and profitability of collusion simultaneously. Under this restriction, it is possible to have a point such as A with a higher discount factor failing to support collusion, while point B with a lower discount factor supports collusion as an equilibrium outcome.

9. CONCLUSION

The purpose of this paper has been to analyze the relationship between the degree of product differentiation and the ability of firms to sustain collusion when firms can redesign their product over time. My model involved a duopoly supergame, in which firms make intertemporal decisions concerning both product choice and price. Due to the role of current product locations as state variables, the supergame possesses a structural time-dependence. When firms behave noncollusively, the game is characterized by a feedback equilibrium which entails a pair of optimal decision rules specifying the equilibrium product locations and a pair of Nash equilibrium prices for each period of the game. I also showed that in this game, a collusive outcome is supported as a closed-loop subgame perfect equilibrium if the firms hold a sufficiently high discount factor.

Two major issues that I initially proposed to address were: 1) how the initial degree of product differentiation affects the ability of firms to collude when they are capable of relocating in the event collusion breaks down, and 2) how the cost of relocation affects the ability of firms to sustain collusion. These questions were answered by examining the set of discount factors that support a joint profit maximum as an equilibrium outcome for some parameter values. Regarding the first issue, it was concluded that collusion is more difficult to sustain for stronger substitutes. This implies that the gains to defecting from a cartel become greater relative to the benefit of remaining in the cartel, as the products become stronger substitutes. From the investigation of the second issue, we learned that collusion is more difficult when the cost of relocation is lower. As the fixed cost of relocation is reduced, firms find it less costly to relocate away from each other in the event that collusion breaks down. Since the post-deviation relocation by the firms reduces the severity of punishment characterized by a price war, it becomes increasingly more difficult to sustain collusion as the fixed cost of relocation is lower.[9]

My result has implications for supergames in more general contexts. The noncooperative theory of collusion tells us that we can achieve a higher degree of collusion as the severity of credible punishment is higher. The main point made in this paper is that this severity of credible punishment is crucially dependent upon the ease with which firms may alter its decision (product design in our case) from one period to the next. If the firms can alter its decision intertemporally with relative ease, severe punishments may no longer be credible. Rather than suffering a severe punishment in the event of the break-down of the cartel, firms would prefer to alter its decision and avoid such punishments. This suggests that an increased flexibility in intertemporal decision making adversely affects the degree of collusion supportable by reducing the severity of credible punishments.

In this paper, the collusive strategies were limited to Friedman-type trigger strategies in which firms permanently revert to the noncollusive equilibrium in the event of defection. Other possibilities include the Most Severe Punishment strategies of Abreu (1986). Future research should consider a wider class of punishment paths.

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[9] An anonymous referee has pointed out to me that these results may hinge upon the two assumptions made in this paper: 1) Relocation cost is fixed and independent of the distance that the firm moves, and 2) each firm produces only a single product in each period in that once a new product is introduced (relocation/redesign), the old product is withdrawn from the market. It is interesting to consider how my results may be modified if these assumptions are altered. Firstly, the relocation cost could be an increasing function of distance moved. Since the firms must now consider the fact that the distance it moves has an impact on its cost, there would be less of an incentive to maximally differentiate one's product from the rivals'. This implies that the equilibrium on the punishment path will generally entail moderate differentiation rather than maximal differentiation. Since moderate differentiation on the punishment path brings about more severe price competition, the punishment is likely to be harsher in the model with variable relocation cost. Consequently, the firms may be able to support a higher degree of collusion if the cost of relocation were to depend on the distance moved. Secondly, the model may involve duopolists who are capable of multi-product operation. Allowing for this possibility introduces two potentially countervailing forces: 1) The incentive to cheat may increase, since the cheater may profitably deviate by introducing a second product which is identical to its rival's product and then engaging in price competition—this may be profitable if the current products are sufficiently differentiated, and 2) the punishment may be severer since the victimized firm may retaliate by introducing a second product itself. Whether or not the potential for multi-product operation strengthens the sustainability of collusion cannot be addressed properly without examining the above factors in detail.

APPENDIX

This appendix contains the proofs of Theorem 2 and Proposition 2.

PROOF OF THEOREM 2. To show that (9) and (10) form a closed-loop subgame perfect equilibrium, we need to show that the strategies form a Nash equilibrium for all histories. When the past history contains a deviation, the strategies dictate $(\phi_1^i(x_1^{t-1}, x_2^{t-1}), \phi_2^i(x_1^{t-1}, x_2^{t-1}); \bar{p}_1(x_1^t, x_2^t), \bar{p}_2(x_1^t, x_2^t))$ be played. Since by Theorem 1 $(\phi_1^i(\cdot), \phi_2^i(\cdot); \bar{p}_1(\cdot), \bar{p}_2(\cdot))$ form a feedback equilibrium in any period t, it is trivial to support it as a closed-loop subgame perfect equilibrium in case of a deviation. If the past history entails mutual collusion in each and every period, the strategies dictate $(\hat{x}, \hat{p})$ be played. In showing that this forms a closed-loop subgame perfect equilibrium, I proceed as follows: 1) describe the feedback equilibria on the punishment path in terms of the discount factor that firms hold, and show its stationarity, 2) examine the profitability of collusion, and 3) find the set of discount factors that support (9) and (10) as a closed-loop subgame perfect equilibrium.

Step 1. Let us examine the feedback equilibria on the punishment path. Since firms have no incentive to deviate in location, the initial product locations for the punishment path are the collusive locations, $(\hat{x}, 1 - \hat{x})$. Given $(\hat{x}, 1 - \hat{x})$ as the current location pair, we may define $\bar{v}(\hat{x})$ and $\underline{v}(\hat{x})$ such that $\bar{v}(\hat{x}) \equiv \bar{v}_1(\hat{x}, 1 - \hat{x}) = \bar{v}_2(\hat{x}, 1 - \hat{x}) = (1/18)b(3\hat{x} + 7\hat{x}^2 - \hat{x}^3)$ and $\underline{v}(\hat{x}) \equiv \underline{v}_1(\hat{x}) = \underline{v}_2(1 - \hat{x}) = (1/18)b(3\hat{x} + 5\hat{x}^2 + \hat{x}^3)$, where $\bar{v}(\hat{x}) \geq \underline{v}(\hat{x})$ for all $\hat{x} \in [0, 1/2)$. Furthermore, we observe that

$$\text{(A.1)} \qquad \underline{v}(\hat{x}) > (1-\delta)f \quad \text{and} \quad \bar{v}(\hat{x}) > (1-\delta)f \qquad \forall\, (\hat{x}, 1-\hat{x}) \in \lambda_1,$$

$$\text{(A.2)} \qquad \underline{v}(\hat{x}) \leq (1-\delta)f \quad \text{and} \quad \bar{v}(\hat{x}) \leq (1-\delta)f \qquad \forall\, (\hat{x}, 1-\hat{x}) \in \lambda_4^1,$$

$$\text{(A.3)} \qquad \underline{v}(\hat{x}) \leq (1-\delta)f \quad \text{and} \quad \bar{v}(\hat{x}) > (1-\delta)f \qquad \forall\, (\hat{x}, 1-\hat{x}) \in \lambda_4^4.$$

Since there exist no symmetric location pairs in the sets, λ_2, λ_3, λ_4^2, and λ_4^3, $\lambda_1 \cup \lambda_4^1 \cup \lambda_4^4$ contains all possible pairs of symmetric product locations. In view of $\bar{v}(\hat{x})$, $\underline{v}(\hat{x})$ and (A.1) through (A.3), one may define $\underline{\delta}$ and $\bar{\delta}$ such that $\bar{v}(\hat{x}) = (1 - \underline{\delta})f$ and $\underline{v}(\hat{x}) = (1 - \bar{\delta})f$. This implies that $\delta \in [\bar{\delta}, 1]$ for $(\hat{x}, 1 - \hat{x}) \in \lambda_1$, $\delta \in (\underline{\delta}, \bar{\delta})$ for $(\hat{x}, 1-\hat{x}) \in \lambda_4^4$, and $\delta \in (0, \underline{\delta}]$ for $(\hat{x}, 1 - \hat{x}) \in \lambda_4^1$. Suppose that firm k defected in price in period s. In period $s + 1$, both firms go on the punishment path by reverting to $(\phi_1^k(\hat{x}, 1 - \hat{x}), \phi_2^k(\hat{x}, 1 - \hat{x}); \bar{p}_1, \bar{p}_2)$, as specified in (9) and (10). By Theorem 1 and the above properties, the feedback equilibrium in $s + 1$ then entails

$$\text{(A.4)} \quad (\phi_1^k(\hat{x}, 1-\hat{x}), \phi_2^k(\hat{x}, 1-\hat{x})) = \begin{cases} (\hat{x}, 1-\hat{x}) & \forall\, \delta \in (0, \underline{\delta}] \\ \left.\begin{matrix} (0, 1-\hat{x}) & \text{if } k = 1 \\ (\hat{x}, 1) & \text{if } k = 2 \end{matrix}\right\} & \forall\, \delta \in (\underline{\delta}, \bar{\delta}) \\ (0, 1) & \forall\, \delta \in [\bar{\delta}, 1]. \end{cases}$$

From Theorem 1, we know that the equilibrium on the punishment path is stationary. The feedback equilibrium as defined in (A.4) will be played in period

$s+1$, and the firms will optimally remain where they are in $s+1$ throughout the remainder of the entire horizon.

Step 2. The next step is to consider the profitability of collusion. Given $(\phi_1^k(\hat{x}, 1-\hat{x}), \phi_2^k(\hat{x}, 1-\hat{x}); \bar{p}_1, \bar{p}_2)$ as the feedback equilibrium on the punishment path, firm i has no incentive to deviate in price if

$$\text{(A.5)} \qquad F(\delta; \hat{x}, f) \equiv [\delta/(1-\delta)][\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(\phi_1^i, \phi_2^i)] + \delta\alpha f \geq v_i^d(\hat{p}; \hat{x}) - \hat{v}_i(\hat{p}; \hat{x}) \equiv G(\hat{x}),$$

where $\alpha = 0$ if $\delta \in (0, \underline{\delta}]$ and $\alpha = 1$ if $\delta \in (\underline{\delta}, 1]$. Noting that (ϕ_1^i, ϕ_2^i) is as defined in (A.4), we may define the functions, $F_1(\delta; \hat{x}, f)$, $F_2(\delta; \hat{x}, f)$, and $F_3(\delta; \hat{x}, f)$ such that

$$\text{(A.6)} \qquad F_1(\delta; \hat{x}, f) \equiv [\delta/(1-\delta)][\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(\hat{x}, 1-\hat{x})] \geq v_i^d(\hat{p}; \hat{x}) - \hat{v}_i(\hat{p}; \hat{x}) \equiv G(\hat{x}),$$

$$\text{(A.7)} \qquad F_2(\delta; \hat{x}, f) \equiv [\delta/(1-\delta)][\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(x_i^*, \hat{x}_k)] + \delta f \geq v_i^d(\hat{p}; \hat{x}) - \hat{v}_i(\hat{p}; \hat{x}) \equiv G(\hat{x}),$$

$$\text{(A.8)} \qquad F_3(\delta; \hat{x}, f) \equiv [\delta/(1-\delta)][\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(0, 1)] + \delta f \geq v_i^d(\hat{p}; \hat{x}) - \hat{v}_i(\hat{p}; \hat{x}) \equiv G(\hat{x}).$$

We observe that $0 = F_i(0; \hat{x}, f) < G(\hat{x}) < \lim_{\delta \to 1} F_i(\delta; \hat{x}, f) = \infty$ for $i = 1, 2, 3$. Furthermore, since $\hat{v}_i(\hat{p}; \hat{x}) > v_i^N(0, 1)$, we observe $\partial F_i(\delta; \hat{x}, f)/\partial\delta > 0$ for all $\delta \in (0, 1)$. Thus, we conclude that there exist δ_1, δ_2, and δ_3 such that $F_i(\delta; \hat{x}, f) \geq G(\hat{x})$ for $\delta \geq \delta_i$, $i = 1, 2, 3$ (Profitability Condition). We also know from (A.4)

$$\text{(A.9)} \qquad F(\delta; \hat{x}, f) = \begin{cases} F_1(\delta; \hat{x}, f) & \forall\, \delta \in (0, \underline{\delta}] \\ F_2(\delta; \hat{x}, f) & \forall\, \delta \in (\underline{\delta}, \bar{\delta}) \\ F_3(\delta; \hat{x}, f) & \forall\, \delta \in [\bar{\delta}, 1). \end{cases}$$

In order for (9) and (10) to be supported as a closed-loop subgame perfect equilibrium, collusion must be enforceable as well as profitable. Thus, not only does the punishment path need to form an equilibrium, the collusive outcome must also be profitable given the credible punishment path. This implies that (A.9) must hold simultaneously with the above profitability condition.

Step 3. Let us define a set of discount factors, $\Delta(\hat{x}, \hat{p}) = e_1 \cup e_2 \cup e_3$, where $e_1 \equiv [\max\{\bar{\delta}, \delta_3\}, 1]$, $e_2 \equiv [\max\{\underline{\delta}, \delta_2\}, \bar{\delta}]$ and $e_3 \equiv [\delta_1, \underline{\delta}]$. Since both $\bar{\delta}$ and δ_3 are strictly less than 1, we know for sure that e_1 is nonempty. Therefore, letting $\hat{\delta} = \max\{\bar{\delta}, \delta_3\}$, we may say that there exists $\hat{\delta}$ such that $(\hat{\delta}, 1) \subset \Delta(\hat{x}, \hat{p})$, and (9) and (10) form a subgame perfect equilibrium for all $\delta \in \Delta(\hat{x}, \hat{p})$. □

PROOF OF PROPOSITION 2. The proof follows from that of Theorem 2. In proving Theorem 2, I defined e_1, e_2, and e_3 such that $e_1 \equiv [\max\{\bar{\delta}, \delta_3\}, 1]$, $e_2 \equiv [\max\{\underline{\delta}, \delta_2\}, \bar{\delta}]$ and $e_3 \equiv [\delta_1, \underline{\delta}]$. In order to show that $\Delta(\hat{x}, \hat{p})$ may be a nonconnected subset of (0, 1) for some f and $(\hat{x}, \hat{p})$, I need only show that e_1 and e_2 are nonempty and may be disjoint in δ for some f and $(\hat{x}, \hat{p})$. I shall prove Proposition 2 through a series of lemmata.

LEMMA A.1. *e_1 is nonempty.*

LEMMA A.2. *e_2 is nonempty if and only if*

$$f \geq \underline{v}(\hat{x}) \frac{[v_i^d(\hat{p}; \hat{x}) - v_i^N(x_i^*, \hat{x}_k) + \underline{v}(\hat{x})]}{[\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(x_i^*, \hat{x}_k) + \underline{v}(\hat{x})]} \equiv \underline{f}(\hat{x}).$$

PROOF. We already know that $\underline{\delta} < \overline{\delta}$. Thus, in order for e_2 to be nonempty, we need only show that $\delta_2 \leq \overline{\delta}$, which is satisfied when $F_2(\overline{\delta}; \hat{x}, f) \geq G(\hat{x})$. Therefore, $\delta_2 \leq \overline{\delta}$ is true if

$$\text{(A.10)} \qquad [\overline{\delta}/(1 - \overline{\delta})][\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(x_i^*, \hat{x}_k)] + \overline{\delta}(f) \geq v_i^d(\hat{p}; \hat{x}) - \hat{v}_i(\hat{p}; \hat{x}),$$
$$\text{where} \quad \overline{\delta} = 1 - (1/f)\underline{v}(\hat{x}).$$

Solving for f, one obtains

$$f \geq \underline{v}(\hat{x}) \frac{[v_i^d(\hat{p}; \hat{x}) - v_i^N(x_i^*, \hat{x}_k) + \underline{v}(\hat{x})]}{[\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(x_i^*, \hat{x}_k) + \underline{v}(\hat{x})]} \equiv \underline{f}(\hat{x}).$$

□

LEMMA A.3. *e_1 and e_2 are disjoint in δ iff $f \in (\underline{f}(\hat{x}), \overline{f}(\hat{x}))$, where*

$$\overline{f}(\hat{x}) \equiv \underline{v}(\hat{x}) \frac{[v_i^d(\hat{p}; \hat{x}) - v_i^N(\hat{x}_i, x_k^*)]}{[\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(\hat{x}_i, x_k^*)]}$$

and

$$\underline{f}(\hat{x}) \equiv \underline{v}(\hat{x}) \frac{[v_i^d(\hat{p}, \hat{x}) - v_i^N(\hat{x}_i, x_k^*) + v_i^N(0, 1) - v_i^N(x_i^*, \hat{x}_k)]}{[\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(\hat{x}_i, x_k^*) + v_i^N(0, 1) - v_i^N(x_i^*, \hat{x}_k)]}.$$

PROOF. e_1 and e_2 are disjoint in δ, if both exist in (0, 1) and $\max\{\delta_3, \overline{\delta}\} = \delta_3$. In such a case, we observe $e_2 = [\max\{\delta_2, \underline{\delta}\}, \overline{\delta}]$ and $e_1 = [\delta_3, 1]$. Notice that there exists a set of discount factors, $(\overline{\delta}, \delta_3)$, in which collusion is not supported at all. Therefore, in order to prove the existence of such a set, we need to show that e_1 and e_2 are nonempty, and $\max\{\delta_3, \overline{\delta}\} = \delta_3$. The existence and nonemptiness of e_2 are guaranteed for $f \geq \underline{f}(\hat{x})$ by Lemma A.2. From (A.8) one sees that $\max\{\delta_3, \overline{\delta}\} = \delta_3$ if and only if $F_3(\overline{\delta}; \hat{x}, f) < G(\hat{x})$:

$$\text{(A.11)} \qquad [\overline{\delta}/(1 - \overline{\delta})][\hat{v}_i(\hat{p}; \hat{x}) - v_i^N(0, 1)] + \overline{\delta}f < v_i^d(\hat{p}; \hat{x}) - \hat{v}_i(\hat{p}; \hat{x}),$$
$$\text{where} \quad \overline{\delta} = 1 - (1/f)\underline{v}(\hat{x}).$$

Solving for f, we obtain $f < \bar{f}(\hat{x})$. Now, we need only show that $\underline{f}(\hat{x}) < \bar{f}(\hat{x})$ to prove that e_1 and e_2 are disjoint in δ for $f \in (\underline{f}(\hat{x}), \bar{f}(\hat{x})]$. Given $\underline{f}(\hat{x})$ and $\bar{f}(\hat{x})$, we observe $\underline{f}(\hat{x}) < \bar{f}(\hat{x})$ if and only if

$$\text{(A.12)} \quad \frac{[v_i^d(\hat{p};\hat{x}) - v_i^N(\hat{x}_i, x_k^*) + v_i^N(0,1) - v_i^N(x_i^*, \hat{x}_k)]}{[\hat{v}_i(\hat{p};\hat{x}) - v_i^N(\hat{x}_i, x_k^*) + v_i^N(0,1) - v_i^N(x_i^*, \hat{x}_k)]} < \frac{[v_i^d(\hat{p};\hat{x}) - v_i^N(\hat{x}_i, x_k^*)]}{[\hat{v}_i(\hat{p};\hat{x}) - v_i^N(\hat{x}_i, x_k^*)]}.$$

Let $A \equiv v_i^d(\hat{p};\hat{x}) - v_i^N(\hat{x}_i, x_k^*)$, $B \equiv \hat{v}_i(\hat{p};\hat{x}) - v_i^N(\hat{x}_i, x_k^*)$, and $C \equiv v_i^N(0, 1) - v_i^N(x_i^*, \hat{x}_k)$. Rewriting (A.12), we obtain $(A + C)/(B + C) < A/B$, where A, B, $C > 0$. By cross-multiplying and simplifying, we can easily see that $\underline{f}(\hat{x}) < \bar{f}(\hat{x})$ if and only if $B < A$. $B < A$ holds trivially since $v_i^d(\hat{p};\hat{x}) > \hat{v}_i(\hat{p};\hat{x})$. Thus, I have just shown that e_1 and e_2 are disjoint in δ for $f \in (\underline{f}(\hat{x}), \bar{f}(\hat{x}))$. □

Since for some $f \in (\underline{f}(\hat{x}), \bar{f}(\hat{x}))$ there exist e_1 and e_2 such that $e_1 \subset \Delta(\hat{x}, \hat{p})$ and $e_2 \subset \Delta(\hat{x}, \hat{p})$, and e_1 and e_2 are disjoint in δ, we may conclude that $\Delta(\hat{x}, \hat{p})$ is a nonconnected subset of (0, 1) for $f \in (\underline{f}(\hat{x}), \bar{f}(\hat{x}))$. This proves Proposition 2. □

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