

## EMERGENT SOCIAL LEARNING NETWORKS IN ORGANIZATIONS WITH HETEROGENEOUS AGENTS

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Received 12 August 2010

Revised 26 January 2011

Two distinct learning mechanisms are considered for a population of agents who engage in decentralized search for the common optimum. An agent may choose to learn via innovation (individual learning) or via imitation (social learning). The agents are endowed with heterogeneous skills in engaging in the two modes of learning. When the agents choose imitation, they also choose whom to learn from. This leads to the *emergence* of a social learning network among agents in the population. This paper focuses on the impact the endowed learning skills have on the individual's choice of learning mechanism as well as the micro and macro structure of the evolving network. Finally, it explores the impact the degree of environmental volatility has on the structure of such networks.

*Keywords:* Social learning networks; innovation; imitation; organizational learning; heterogeneous agents.

### 1. Introduction

It has been noted that an individual embedded in a social system may employ two distinct modes of learning in solving problems — innovation (individual learning) and imitation (social learning):

Sometimes scientists modify their cognitive states as results of *asocial* interactions, sometimes they change their minds through *social* exchanges. The obvious exemplars for the former are the solitary experimentalist at work with apparatus and samples and the lone field observer attending to the organisms . . . Paradigm cases of conversations with peers are those episodes in which one scientist is told something by another (and believes it) or when a change in commitment is caused by the reading of a text. The point of the distinction is evidently to separate those episodes that (very roughly) consist in finding things out for oneself from those in which one relies on others . . . [8, p. 60]

Does a high level of social knowledge require a high level of social interaction? . . . In principle an impressive aggregate knowledge might be acquired if each member independently explores and discovers the facts of interest. A hallmark of human culture, however, is to enhance the social fund of knowledge by sharing discovered facts with one another. [7, p. 103]

The issue of individual versus social learning is central to the long-term performance of social systems such as business organizations and teams engaged in recurrent problem-solving with multiple agents. Effective learning at the organizational level requires both individual learning *and* social learning at the agent level: Individual learning by an agent increases the degree of variation in the set of ideas available for adoption (thereby improving the organization's ability to adapt to changes in the long run), while social learning facilitates the organization-wide diffusion of those ideas that are already proven useful (thereby improving the organization's performance in the short run). The two modes of learning are, hence, complements at the organizational level.

While the intuition behind the complementarity among the two learning modes is rather straightforward, understanding the extent to which it is realized at the organizational level requires a careful examination of the following three issues. First, how is this complementary relationship realized in a decentralized organization where the individual agents choose the learning modes *autonomously* and *in parallel*?<sup>a</sup> Second, when social learning is chosen by an individual, that individual will need to identify whom she will learn from, and this is determined by the social network she possesses at that point. This social network is likely to evolve over time, however, as each individual adjusts the likelihood of returning to a given person for learning on the basis of the success or failure of her decision to learn from that person. What kinds of networks are likely to develop as the consequence of this process? Third, when learning takes place in a dynamic environment, how does the extent of environmental turbulence affect which learning mechanism individuals choose over time and what types of structure will the learning networks evolve to attain? This paper explores these issues by developing an agent-based computational model of the decentralized process by which ideas are generated by

<sup>a</sup>To see the relevant forces behind this issue, observe that an individual with limited time resource and cognitive capacity can only pursue one mode of learning at any given point in time and, hence, must choose how to learn. Given the group of agents who must make autonomous choices between the two modes of learning, the organization then faces two unintended consequences (trade-offs): (i) Since individual learning and social learning are substitutes at the agent level, the pursuit of one learning mode by an individual comes at the cost of foregone benefits realizable from using the other mode (both for the individual and for the organization); (ii) Diffusion of a successful idea, while improving the short-run performance of the individual and the organization, tends to reduce the degree of variation in the existing pool of ideas, thereby weakening the organization's ability to adapt in the long run. In a dynamic environment in which learning must go on continually, the exact manner in which these trade-offs affect the organizational performance will depend on the degree of environmental turbulence.

individual agents (through individual learning) and diffused in the organization (through social learning).

The model entails a population of myopic, though adaptive, agents searching for a common optimum (organizational goal) in the space of possible things that one can do. The agents choose whether to allocate their efforts to discovering new ideas — innovation — or to observing the ideas of others — imitation. When they engage in imitation, agents decide *whom to learn from*, which takes the form of establishing links in a social network in terms of how observation probabilities are distributed across individuals. These probabilities are then adjusted by the individuals over time via reinforcement learning. The process of knowledge creation and diffusion occurs in the context of a changing environment as represented by stochastic movement in the common optimum. Whether or not an individual's effort to innovate or imitate is productive depends on whether his or her inherent ability lies in generating new ideas or in establishing communication links with other agents. The agents are assumed to be heterogeneous in these capabilities. Since the knowledge transfer in our organization is carried out through a purely decentralized process with no centralized coordination, this is a model of *informal* organizational learning.

Note that there are two distinct stages to individual decision making in the proposed model. The first stage looks at the choice between individual and social learning. The second stage, which becomes relevant only when social learning is chosen in the first stage, addresses the individual's choice of *whom* to observe. Given the endowed skill differentials (i.e. innovativeness versus connectivity) among individuals, I then address a series of issues involving the choices made at the individual level in terms of learning mode and the consequent outcomes at the organizational level as implied by the endogenous structure of the social learning network. More specifically, I ask: How does agent heterogeneity in learning skills feed into the private choices they make in terms of allocating their efforts between individual learning (innovation) and social learning (imitation through social interactions)? Do agents with higher absolute innovation (imitation) skills necessarily choose to engage in innovation (imitation) with a greater probability? Do highly innovative individuals perform better than less innovative individuals? In the course of engaging in social learning, individuals get to develop informal knowledge networks — who learns from whom — within the organization. What are the structural properties of the networks thus developed? How does the exact combination of the learning skills endowed by each individual determine the emergent structure of the social learning network? How are these relationships affected by the degree of volatility in the environment?

The next section provides a brief review of the related literature. The formal model is then presented in Sec. 3. Section 4 describes the design of computational experiments performed in the paper. How the heterogeneity in learning skills affects the steady-state choices between innovation and imitation is discussed in Sec. 5.

Section 6 provides a detailed analysis of the structure and performance of the emergent network. Section 7 offers concluding remarks.

## 2. Review of the Literature

How the individuals' decisions to engage in innovation or imitation affect an organization is closely related to the force identified in the organizational learning literature as *exploration* versus *exploitation* trade-off. [9], a classic paper in this line of research, considers the role of an organizational code that adapts over time and determines the relative rates of exploration and exploitation. The agents in the organization learn *from* the code, but the code itself also *adapts to* the beliefs of better-performing agents. In that model, the "organizational code" is a device that is exogenously specified. Instead of assuming the existence of a "code," my model replaces it with a social learning network which is endogenously developed through the dynamic choices of learning modes made by the individual agents — i.e., agents learn directly from one another rather than indirectly from the organizational code. It, thus, provides a detailed look at the very mechanism that brings about the organizational code. The exploration/exploitation trade-off is also examined in Ref. 10 though their focus is on the role of the formal hierarchy (in the allocation of authority) in problem-solving. I replace the formal hierarchy with its fixed links with an informal endogenous network and thus address a different set of questions.

The model in this paper is also closely related to the one used in Refs. [4] and [5]. The objective of [4] was to characterize network structure and population performance and explore their dependence on the reliability of the communications technology, as well as the innovativeness of agents. Contrary to the model in this paper, [4] assumed agents with homogeneous learning skills — i.e. all agents had the equal level of imitation capability (determined by the communication technology available to all) and of innovation capability. When the communication technology is poor, it was found, not surprisingly, that technological improvements enhance performance. What was surprising was that if the communication technology is sufficiently effective, further improvements are detrimental. Better communications allows more social learning among agents, which results in agents having very similar solutions. The ensuing lack of diversity within the social network meant that the population of agents is ill-equipped to adapt to a changing environment. Thus, a better communications technology can lead to too structured a network from the perspective of promoting innovation. The detailed structural properties of the network, however, were not examined in that paper and that is what I intend to explore in this paper.

[5] allows agents with heterogeneous learning skills, but only in a limited way. It considers a population that is comprised of three types: *Innovators* who are highly productive in generating new ideas, *Imitators* who are highly productive in identifying and copying the ideas of others, and *Regular Agents* who are moderately productive at both activities. Those individuals belonging to a given type have

common learning skills. In this framework, the study investigated the architecture of the networks that evolve and how the emergence of connectors depends on the distribution of the types as defined above. The present paper generalizes this approach by allowing the learning skills of the agents to be uniformly heterogeneous in both dimensions. Furthermore, I attain a richer set of results through a detailed examination of the evolved imitation probabilities.

Finally, inter-personal learning networks have been explored empirically in the context of organizational and team learning. [6] takes a qualitative look at the information flow among executives in a large organization, using a social network analysis. Partly based on the results from [2], [6] proposes a formal model of information-seeking behavior in the social learning process and statistically tests the significance of several relational characteristics that facilitate such behavior. [12] examines the relationship between the centrality of an individual in advice network and his/her job performance. They find that centrality is positively related to performance. As the authors recognize, however, their study can not rule out the possibility that “coworkers seek out high performers as sources of advice, thus enhancing high performers’ central positions within informal networks.” [12, p. 323] This possibility is the main feature of the model presented in this paper, as it endogenizes individual agents’ choice of *whom* to imitate through a reinforcement learning mechanism based on their past successes.

### 3. The Model

#### 3.1. Agents, tasks, goal and performance

The organization consists of  $L$  individuals. Each individual engages in an operation which can be broken down into  $H$  separate tasks. There are several different methods which can be used to perform each task. The method chosen by an agent for a given task is represented by a sequence of  $d$  bits (0 or 1) such that there are  $2^d$  possible methods available for each task. Let  $z_i^h(t)$  denote the method used by individual  $i$  in task  $h$  in period  $t$ . In any period  $t$ , an individual  $i$  is then fully characterized by a binary vector of  $H \cdot d$  dimensions, which I denote by  $\underline{z}_i(t)$ , where  $\underline{z}_i(t)$  is a connected sequence of methods,  $z_i^1(t), z_i^2(t), \dots$ , and  $z_i^H(t)$  — one method (a string of  $d$  bits) for each task. To be more concrete, consider an operation having five separate tasks with four dimensions to each task so that  $H = 5$  and  $d = 4$ :

Task ( $h$ ) :	#1	#2	#3	#4	#5
Methods ( $\underline{z}_i^h(t)$ ) :	1001	1101	0001	1010	0101

There are sixteen ( $=2^4$ ) different methods for each task. Since the operation is completely described by a vector of 20 ( $=5 \times 4$ ) bits, there are  $2^{20}$  possible bit configurations (i.e. methods vectors) for the overall operation.

The degree of heterogeneity between two methods vectors,  $\underline{z}_i$  and  $\underline{z}_j$ , is measured using “Hamming distance” which is defined as the number of positions for which the corresponding bits differ. I shall denote it by  $D(\underline{z}_i, \underline{z}_j)$ .

In period  $t$ , the population faces a common goal vector,  $\widehat{z}(t)$ , which is also a binary vector of  $H \cdot d$  dimensions. The degree of turbulence in task environments is captured by intertemporal variability in  $\widehat{z}(t)$ , the details of which are explained in Sec. 3.4.

The individuals are uninformed about the goal vector  $\widehat{z}(t)$  *ex ante*, but engage in “search” to get as close to it as possible. Given  $H$  tasks with  $d$  bits in each task and the goal vector  $\widehat{z}(t)$ , the period- $t$  performance of individual  $i$  is then measured by  $\pi_i(t)$ , where

$$\pi_i(t) = H \cdot d - D(z_i(t), \widehat{z}(t)). \quad (1)$$

Hence, the performance of agent  $i$  is greater as the Hamming distance to the goal vector is shorter. It reaches its maximum value of  $H \cdot d$  when agent  $i$  fully attains its goal such that  $z_i(t) = \widehat{z}(t)$ .

### 3.2. Modeling innovation and imitation

In a given period, an individual’s search for the current optimum is carried out through two distinct mechanisms, innovation and imitation. Innovation occurs when an individual independently discovers and considers for implementation a random method for a randomly chosen task. Imitation is when an individual selects someone (probabilistically) in the organization and then observes and considers implementing the method currently deployed by that agent for one randomly chosen task.<sup>b</sup>

Whether obtained through innovation or imitation, an experimental method is actually adopted if and only if its adoption brings the agent closer to the goal by decreasing the Hamming distance between the agent’s new methods vector and the goal vector. For clarity, let us consider the following example with  $H = 5$  and  $d = 2$ :

$$\begin{array}{l} \text{common goal vector:} \quad 01 \mid 10 \mid 10 \mid 01 \mid 01 \\ \text{agent } i\text{'s current methods vector:} \quad 01 \mid 01 \mid 11 \mid 00 \mid 11 \end{array}$$

The relevant operation has five tasks. In each task, there are four distinct methods that can be tried: (0,0), (0,1), (1,0), and (1,1). Agent  $i$  with the above current methods vector is then employing the method (0,1) for task 1, (0,1) for task 2, (1,1) for task 3, (0,0) for task 4, and (1,1) for task 5. The Hamming distance between  $i$ ’s current methods vector and the goal vector is five. Suppose  $i$  chooses to innovate in task 1. For task 1, she randomly selects a method from the set of all

<sup>b</sup>This process of social learning shares some common features with the process of cultural change modelled in Ref. [1]. One of the determinants considered in that paper is the scope of cultural possibilities which include the number of cultural features and the number of possible traits that each feature can have. While it is possible to explore a similar issue in my model by varying the number of tasks and the number of bits per task, it is beyond the current scope of the paper and is left for future research.

available methods,  $\{(0, 0), (0, 1), (1, 0), (1, 1)\}$ . Let us assume that she comes up with the idea of  $(1, 1)$  for task 1. The experimental methods vector for agent  $i$  is then

$$\text{agent } i\text{'s experimental methods vector: } 11 \mid 01 \mid 11 \mid 00 \mid 11$$

where the method  $(0, 1)$  in task 1 is replaced with  $(1, 1)$ . This raises the Hamming distance to the goal vector from five to six and, hence, is rejected by the agent. Alternatively, suppose that agent  $i$  chooses to imitate and ends up observing the method used for task 4 by another agent  $j$  ( $\neq i$ ) whose methods vector is

$$\text{agent } j\text{'s current methods vector: } 10 \mid 10 \mid 11 \mid 01 \mid 01$$

Since  $j$ 's method in task 4 is  $(0, 1)$ , when it is tried by agent  $i$ , her experimental methods vector becomes

$$\text{agent } i\text{'s experimental methods vector: } 01 \mid 01 \mid 11 \mid 01 \mid 11$$

which then reduces the Hamming distance to the goal vector to four, hence, the experimental methods vector becomes  $i$ 's new methods vector.

### 3.3. Endogenizing choices for innovation and imitation

I assume that in each period an individual may engage in *either* innovation *or* imitation by using the network. How exactly does an individual choose between innovation and imitation and, if he chooses to imitate, how does he decide whom to imitate? I model this as a two-stage stochastic decision process with reinforcement learning. Figure 1 describes the timing of decisions in my model. In stage 1 of

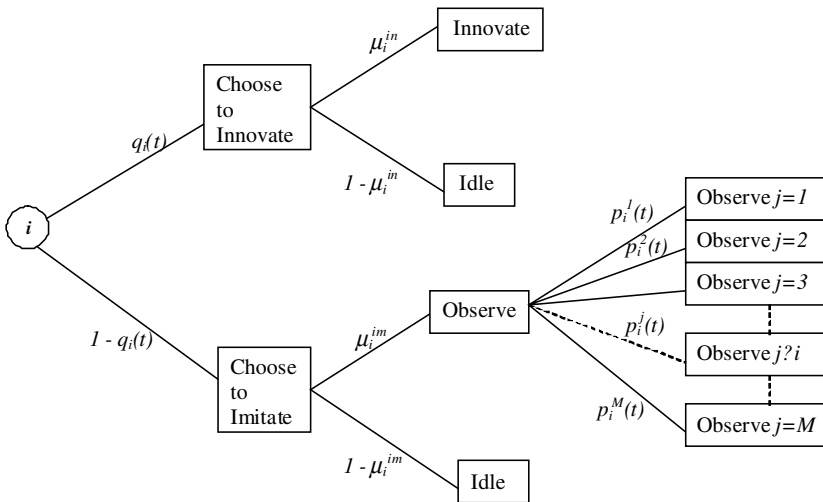


Fig. 1. Decision sequence of individual  $i$  in period  $t$ .

period  $t$ , individual  $i$  is in possession of the current methods vector,  $\underline{z}_i(t)$ , and chooses to innovate with probability  $q_i(t)$  and imitate with probability  $1 - q_i(t)$ . If the agent chooses to innovate then, with probability  $\mu_i^{im}$ , he or she generates an idea which is a randomly chosen task  $h \in \{1, \dots, H\}$  and a randomly chosen method for that task such that the experimental method vector, denoted  $\underline{z}'_i(t)$ , has the same methods as  $\underline{z}_i(t)$  in all tasks except for the chosen task  $h$ . The method for the chosen task  $h$  will be replaced with the randomly chosen method, as explained in the example provided in the previous sub-section. This experimental vector is adopted by  $i$  if and only if its adoption decreases the Hamming distance between the agent and the current goal vector,  $\widehat{z}(t)$ , in which case the methods vector in period  $t+1$  is the experimental vector,  $\underline{z}'_i(t)$ .<sup>c</sup> Otherwise, the experimental vector is discarded and the methods vector in  $t+1$  is the same as  $\underline{z}_i(t)$ . Alternatively, when the individual fails to generate an idea, which occurs with probability  $1 - \mu_i^{im}$ , the methods vector in  $t+1$  remains the same as  $\underline{z}_i(t)$ .

Now suppose individual  $i$  chooses to imitate in stage 1. Given that the agent decides to imitate someone else, he or she taps into the network to make an observation. Tapping into the network is also a probabilistic event, in which with probability  $\mu_i^{im}$  the agent is connected to the network, while with probability  $1 - \mu_i^{im}$  the agent fails to connect. An agent who is connected then enters stage 2 of the decision process in which he or she must select another agent to be studied for possible imitation. Let  $p_i^j(t)$  be the probability with which  $i$  observes  $j$  in period  $t$  so  $\sum_{j \neq i} p_i^j(t) = 1$  for all  $i$ . If agent  $i$  observes another agent  $l$ , that observation involves a randomly chosen task  $h$  and the current method used by agent  $l$  in that task,  $\underline{z}_l^h(t)$ . Let  $\underline{z}''_i(t)$  be the experimental vector such that it has the same methods as in  $\underline{z}_i(t)$  for all tasks except for task  $h$ , and the method in  $h$  is replaced with  $\underline{z}_l^h(t)$ . Adoption or rejection of the observed method is based on the Hamming distance criterion such that it is adopted if and only if it reduces the Hamming distance to the goal vector  $\widehat{z}(t)$ : the new methods vector in  $t+1$  is, hence, the experimental vector,  $\underline{z}''_i(t)$ , in the case of adoption. Otherwise, it remains the same as  $\underline{z}_i(t)$ . Again, if the agent fails to connect to the network, which occurs with probability  $1 - \mu_i^{im}$ , the new methods vector remains the same as  $\underline{z}_i(t)$ .

The probabilities,  $q_i(t)$  and  $\{p_i^1(t), \dots, p_i^{i-1}(t), p_i^{i+1}(t), \dots, p_i^L(t)\}$ , are adjusted over time by individual agents according to a reinforcement learning rule.<sup>d</sup> I adopt a version of the *Experience-Weighted Attraction* (EWA) learning rule as described

<sup>c</sup>I am, hence, assuming that the agents have complete information about the performance level associated with the proposed method vector. This is not meant to represent the reality, as I believe the human decision-makers are often uncertain as to how the proposed method will perform — i.e. they often err and adopt methods that are poorly suited for the tasks in hand. Nevertheless, it seems likely that they will engage in short-term experimentations (either in their head or in their lab) in order to improve the precision of their evaluation. By specifying complete information, I am essentially assuming that these experimentations are done instantaneously and costlessly. With this simplification, I avoid overloading the model which is already complex.

<sup>d</sup>See [13] for a general discussion of reinforcement learning mechanisms.



in [3]. Under this rule, an agent has a numerical attraction for each possible action. The learning rule specifies how attractions are updated by the agent's experience and how the probabilities of choosing different actions depend on attractions. The main feature of the rule is that *a positive outcome realized from a course of action reinforces the likelihood of that same action being chosen again*.

Using the EWA-rule,  $q_i(t)$  is adjusted each period on the basis of evolving attraction measures,  $A_i^{in}(t)$  for innovation and  $A_i^{im}(t)$  for imitation. The following process drives the evolution of  $A_i^{in}(t)$  and  $A_i^{im}(t)$ . If the agent chose to pursue *Innovation* and discovered and then adopted the new idea, the attraction measure for *Innovation* increases by 1 — i.e.,  $A_i^{in}(t+1) = A_i^{in}(t) + 1$ . If the agent chose to innovate but was unsuccessful (either because he or she failed to generate an idea, or because the idea generated was not useful) or if the agent instead chose to imitate, then the attraction measure for innovation is simply the attraction level from the previous period — i.e.,  $A_i^{in}(t+1) = A_i^{in}(t)$ .<sup>e</sup> Similarly, a success or failure in imitation at  $t$  has the identical influence on  $A_i^{im}(t+1)$  such that  $A_i^{im}(t+1) = A_i^{im}(t) + 1$  if  $i$  adopted a method through imitation in  $t$ , while  $A_i^{im}(t+1) = A_i^{im}(t)$ , otherwise. Given  $A_i^{in}(t)$  and  $A_i^{im}(t)$ , one derives the choice probability of innovation in period  $t$  as follows:

$$q_i(t) = \frac{A_i^{in}(t)}{A_i^{in}(t) + A_i^{im}(t)}. \quad (2)$$

The probability of imitation is, of course,  $1 - q_i(t)$ . The expression in (2) says that a favorable experience through innovation (imitation) raises the probability that an agent will choose to innovate (imitate) again in the future.

The stage-2 attractions and the probabilities are derived similarly. Let  $B_i^j(t)$  be agent  $i$ 's attraction to another agent  $j$  in period  $t$ . Its evolution follows the same rule as that of  $A_i^{in}(t)$  and  $A_i^{im}(t)$ , in that  $B_i^j(t+1) = B_i^j(t) + 1$  if agent  $i$  successfully imitated another agent  $j$  in  $t$ , while  $B_i^j(t+1) = B_i^j(t)$ , if otherwise. The probability that agent  $i$  observes agent  $j$  in period  $t$  is adjusted each period on the basis of the attraction measures,  $\{B_i^j(t)\}_{j \neq i}$ :

$$p_i^j(t) = \frac{B_i^j(t)}{\sum_{h \neq i} B_i^h(t)} \quad (3)$$

for all  $i$  and for all  $j \neq i$ . Agent  $i$ 's success in imitating another agent  $j$  then further raises the probability that the same agent will be observed again relative to others.

There are two distinct sets of probabilities in this model. One set of probabilities,  $q_i(t)$  and  $\{p_i^j(t)\}_{j \neq i}$ , are endogenously derived and evolve over time in response to the personal experiences of agent  $i$ . Another set of probabilities,  $\mu_i^{in}$  and  $\mu_i^{im}$ , are

<sup>e</sup>There is actually a decay factor of  $\phi$  in the equations of motion for the attractions such that  $A_i^{in}(t+1) = \phi A_i^{in}(t) + 1$  or  $\phi A_i^{im}(t)$ . It is for the analytical simplicity that I assume  $\phi = 1$  (no decay) in my work. The same goes for  $A_i^{im}(t)$  and  $B_i^j(t)$ 's.

exogenously specified and are imposed on the model as parameters. They control the capabilities of individual agents to independently innovate or to imitate someone else in the organization via social learning. I will refer to them jointly as “agent  $i$ ’s endowed learning skills.”

### 3.4. *Modeling turbulence in task environment*

If the organization faced one fixed problem then all agents will eventually attain the global optimum through the search process described in the previous section. In such a case, the measure of performance for an individual is the speed with which the goal is achieved. In reality, however, most business organizations face a series of related problems, since the current problem they are working on may change due to a number of market factors such as the actions of competing firms, technological advances in another industry, or intertemporal changes in customer preferences. Rather than model agents as facing a fixed problem, I choose to model them as facing a series of related problems. For analytical tractability, this is done by allowing the problem itself to evolve stochastically over time. Performance of an individual then depends not just on the speed with which a problem is solved, but also on how well he or she responds to an evolving environment.

Change or turbulence is specified in the model by first assigning an initial goal vector,  $\hat{z}(0)$ , to the organization and then specifying a dynamic process by which it shifts over time. In period  $t$ , all agents have the common goal vector of  $\hat{z}(t)$ . In period  $t+1$ , the goal changes with probability  $\sigma$  and stays the same with probability  $(1 - \sigma)$ . The shift dynamic of the goal vector is guided by the following stochastic process. The goal in  $t+1$ , if different from  $\hat{z}(t)$ , is then chosen *iid* from the set of points that lie within the Hamming distance  $\rho$  of  $\hat{z}(t)$ . The goal vector for the organization then stochastically shifts while remaining within Hamming distance  $\rho$  of the *current* goal. This allows us to control the possible size of the inter-temporal change. The greater is  $\sigma$  and/or  $\rho$ , the more frequent and/or variable is the change, respectively, in the organization’s goal vector.

## 4. Design of Computational Experiments

The underlying simulation model specifies  $H = 24$  and  $d = 4$ , so that there are 96 total bits in a methods vector and over  $7.9 \times 10^{28}$  ( $\cong 2^{96}$ ) possibilities in the search space. For the organization, I consider a population of one hundred and fifty individuals:  $L = 150$ . These individuals are assumed to have heterogeneous learning skills such that  $\mu_i^{in}$  and  $\mu_i^{im}$  are independent random draws from a uniform distribution over  $[0, 1]$  and, once chosen, they remain fixed over the entire horizon.

I assume that the *initial* practices of the agents are completely homogeneous so that  $\underline{z}_i(0) = \underline{z}_j(0) \forall i \neq j$ . This is to ensure that any social learning (imitation) occurring over the horizon under study entails only newly generated knowledge. Otherwise, the initial variation in the information levels of the agents will induce

some imitation activities, introducing unnecessary random noise into the system. The common initial methods vector is assumed to be an independent draw from  $\{0, 1\}^{Hd}$ .

The parameters affecting the endogenous variables are  $\sigma$  and  $\rho$  — the frequency and magnitude of the environmental changes for the organization. I consider values of  $\sigma$  from  $\{0.1, 0.2, 0.3, 0.5\}$  and  $\rho$  from  $\{1, 3, 5, 9\}$ .

The initial attraction stocks are set at  $B_i^j(0) = 1$  for all  $i$  and for all  $j \neq i$ , and  $A_i^{in}(0) = A_i^{im}(0) = 1$  for all  $i$ . Hence, an individual, in  $t = 0$ , is equally likely to engage in innovation and imitation —  $q_i(0) = 0.5$  — and has no inclination to observe one individual over another *ex ante* — i.e.  $p_i^j(0) = \frac{1}{L-1}$  ( $=\frac{1}{149} \cong 0.0067$  in our experiments) for all  $i$  and for all  $j \neq i$ .

All computational experiments carried out here assume a horizon of 15,000 periods. The time-series of the performance measures are observed to reach a steady-state by the 2,000th period.<sup>f</sup> I measure the steady-state performance of individual  $i$ , denoted  $\bar{\pi}_i$ , to be the average over the last 5,000 periods of this horizon so that  $\bar{\pi}_i = \frac{1}{5,000} \sum_{t=10,001}^{15,000} \pi_i(t)$ . Likewise, the steady-state values of other endogenous variables in the study are also computed as the average over the last 5,000 periods and denoted  $\bar{q}_i$  and  $\bar{p}_i^j$ . Finally, all of the experiments were based on 100 replications, each using a fresh set of random numbers.<sup>g</sup>

Tables 1 and 2 summarize the parameters and the endogenous probabilities discussed in this section.

Table 1. List of parameters.

Notations	Definitions	Parameter values considered
$L$	No. of agents in the organization	150
$H$	No. of separate tasks	24
$d$	No. of dimensions per task	4
$\sigma$	Probability that the goal vector changes from $t$ to $t + 1$	$\{.1, .2, .3, .5\}$
$\rho$	Maximum no. of dimensions in the goal vector that can change from $t$ to $t + 1$	$\{1, 3, 5, 9\}$
$\mu_i^{in}$	Probability that agent $i$ generates an idea in any given period	$[0, 1]$
$\mu_i^{im}$	Probability that agent $i$ taps into its network to imitate another agent	$[0, 1]$
$A_i^{in}(0)$	Agent $i$ 's attraction for innovation in $t = 0$	1
$A_i^{im}(0)$	Agent $i$ 's attraction for imitation in $t = 0$	1
$B_i^j(0)$	Agent $i$ 's attraction to agent $j$ in $t = 0$	1

<sup>f</sup>By a “steady-state,” I mean the state in which the *mean* value of the variable — i.e. mean across multiple replications — is independent of time. This is to be contrasted to “transient” periods, in which the mean value of the variable changes over time (presumably on its way to converge on some steady-state).

<sup>g</sup>A replication is the running of the model for 15,000 periods given a set of random numbers. For each parameter configuration considered in this paper, the model is then run for a total of 1.5 million periods (15,000 periods per replication  $\times$  100 independent replications).

Table 2. List of endogenous probabilities.

Notations	Definitions
$q_i(t)$	Probability that agent $i$ chooses to innovate in $t$
$1 - q_i(t)$	Probability that agent $i$ chooses to imitate in $t$
$p_i^j(t)$	Probability that agent $i$ observes agent $j$ in $t$

## 5. Endogenous Choice of Innovation versus Imitation and the Steady-State Performance of the Agents: Division of Cognitive Labor and Specialization

### 5.1. Baseline analysis

For the baseline analysis, I specify  $\sigma = 0.1$  and  $\rho = 1$ . Hence, there is a probability of 0.1 that the environment will change from  $t$  to  $t + 1$ . If the environment changes, it involves a change in only one randomly chosen task.

I first ask what choices are made in the steady-state by the agents in terms of the learning mechanism — i.e. stage-1 choice between innovation (individual learning) and imitation (social learning). I then ask how these choices are affected by the innate learning skills of the agents,  $(\mu_i^{in}, \mu_i^{im})$ . Do agents with superior ability to innovate (imitate) necessarily choose innovation (imitation) with a greater probability? Remember that innovation and imitation are alternatives that compete directly against each other for agents' time and effort. If an agent chooses to innovate, it comes at the expense of imitation. In order to answer this question, I first look at the steady-state probability of innovation,  $\bar{q}_i$ , for all agents. Since there are 150 agents per replication and I run 100 independent replications using a fresh set of random numbers for each replication, I have a total of 15,000 observations on  $\bar{q}_i$ .<sup>h</sup> For each observation of  $\bar{q}_i$  there is a  $(\mu_i^{in}, \mu_i^{im})$  pair, which is the innovation and imitation capabilities that agent  $i$  is endowed with. In Fig. 2, I plot the magnitude of  $\bar{q}_i$  as the gray-level of a point in the two-dimensional probability space, in which the horizontal and vertical coordinates capture the values of  $\mu_i^{in}$  and  $\mu_i^{im}$ , respectively. The lighter the gray-level of the dot, the higher is the value of  $\bar{q}_i$ . A distinct pattern emerges in this figure: an agent with a sufficiently high (low) level of  $\mu_i^{im}$  tends to choose imitation with a high (low) probability — i.e.,  $\bar{q}_i$  is lower. It is also notable that for a given level of  $\mu_i^{in}$  the value of  $\bar{q}_i$  drops rather abruptly at some critical value of  $\mu_i^{im}$  as  $\mu_i^{im}$  is raised. The critical value of  $\mu_i^{im}$  where such transition occurs appears to increase in  $\mu_i^{in}$ . The abrupt transition in the values of  $\bar{q}_i$ , as captured in Fig. 2, indicates the following property.

<sup>h</sup>I am then pooling observations from 100 independent replications. A detailed look at the observations from each replication indicates that the properties I identify for the pooled observations are also present for each individual replication. Hence, I lose no information by pooling these observations.

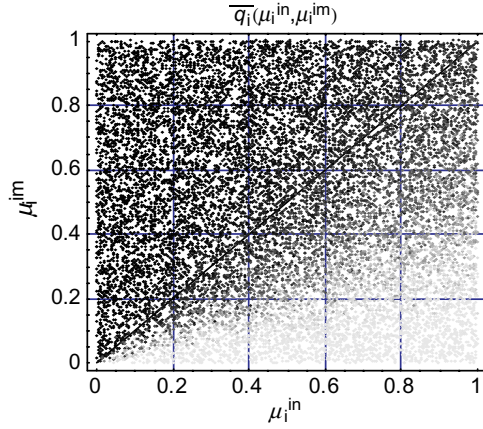


Fig. 2. Choice probabilities of innovation ( $\sigma = 0.1; \rho = 1$ ).

**Property 1.** The population bifurcates over time into two distinct groups in terms of their choice of a learning mode; one group focusing mostly on innovating and the other group focusing mostly on imitating the existing ideas through their networks.

The extent of such bifurcation is seen clearly in Fig. 3, in which I divide the probability range of  $[0, 1]$  into ten bins of equal size and report the percentage of the population (15,000 observations) who have  $\bar{q}_i$  in each bin. As can be seen, over 40% of the population choose innovation with a probability less than 0.1 (and, hence, choose imitation with a probability greater than 0.9), while over 15% choose

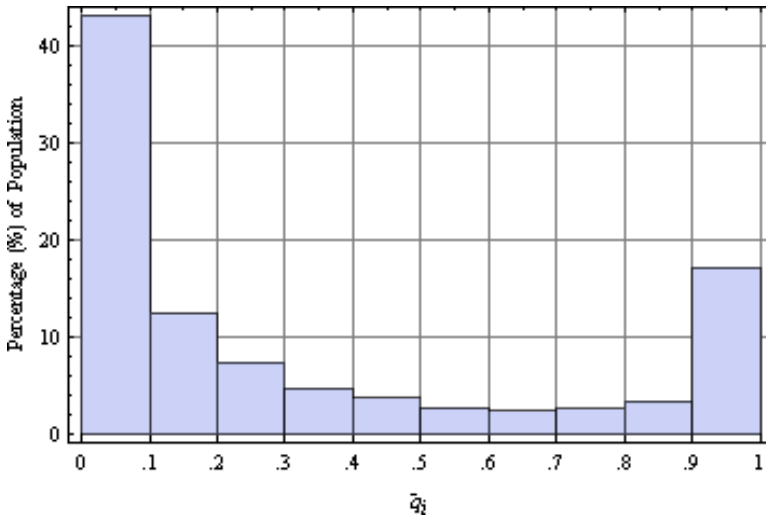


Fig. 3. Distribution of  $\bar{q}_i$ s ( $\sigma = 0.1; \rho = 1$ ).

innovation with a probability greater than 0.9. The rest of the population is divided rather evenly between the two extreme peaks. It is clear that the majority of agents tend toward either innovating or imitating full time rather than alternating evenly between the two learning mechanisms.

Which mode of learning an individual concentrates on depends partly on his/her endowed learning skills — as can be seen by the obvious choices made by those with relative skills in innovation (high  $\mu_i^{in}$  but low  $\mu_i^{im}$ ) and those with skills in imitation (low  $\mu_i^{in}$  but high  $\mu_i^{im}$ ). Just as significant and far more interesting, however, is the dramatic contrast in the innovation probabilities held by those agents whose skill levels in innovation and imitation are not so different — i.e. those near the diagonal in Fig. 2. Two individuals with very similar endowments in learning skills evolve to focus on very different modes of learning. For these individuals, the eventual choice of a learning mode is then less *skill-based* and more *interaction-based*: The short-term attraction to an agent of imitation through network depends on there being sufficient amount of discoveries to go around (determined by how many others are currently innovating), while the long-term attraction to an agent of innovation is influenced by the extent to which imitation serves as a viable alternative to innovation (and this is also determined by the choices made by others in the population). The result is an endogenously attained *division of cognitive labor* and *specialization*, in which a group of individuals specialize in generating new ideas while the rest of the population *free-ride* on these discoveries by directly copying them through the endogenous networks they develop amongst themselves. As the value of the network learning and imitation is limited by the extent of the fresh (non-redundant) ideas in the system, it also means that those agents with even a mild advantage in innovation will evolve toward exclusively generating ideas and, hence, the eventual bifurcation of the population.<sup>i</sup>

Given the specialization effect observed in Figs. 2 and 3, what is most striking is the fact that a large proportion of the individuals in the population learn through imitation rather than innovation — over 70% of the population has  $0 \leq \bar{q}_i \leq 0.5$ . Furthermore, it turns out that the steady-state performance levels of the agents depend more on their abilities to imitate than on their abilities to innovate. To see this, I collect the steady-state performance levels for all 150 agents from the 100 independent replications. This yields a total of 15,000 observations on  $\bar{\pi}_i$ , each coupled with a pair of values for  $(\mu_i^{in}, \mu_i^{im})$  that are specific to agent  $i$ . The contour plot of the resulting performance surface is shown in Fig. 4.

The plot shows that an agent's performance increases in  $\mu_i^{im}$  for all values of  $\mu_i^{in}$ , while the impact of  $\mu_i^{in}$  on the performance tends to be non-monotonic for some values of  $\mu_i^{im}$ . Hence, the imitation capability is valuable for all types of agents. The same can not be said of the innovation capability. A greater innovation capability

<sup>i</sup>It is interesting to note that the endogenous specialization and the bifurcation of the population that arise in my model resembles the multiplicity of asymmetric Nash equilibria that can arise in some game-theoretic models.

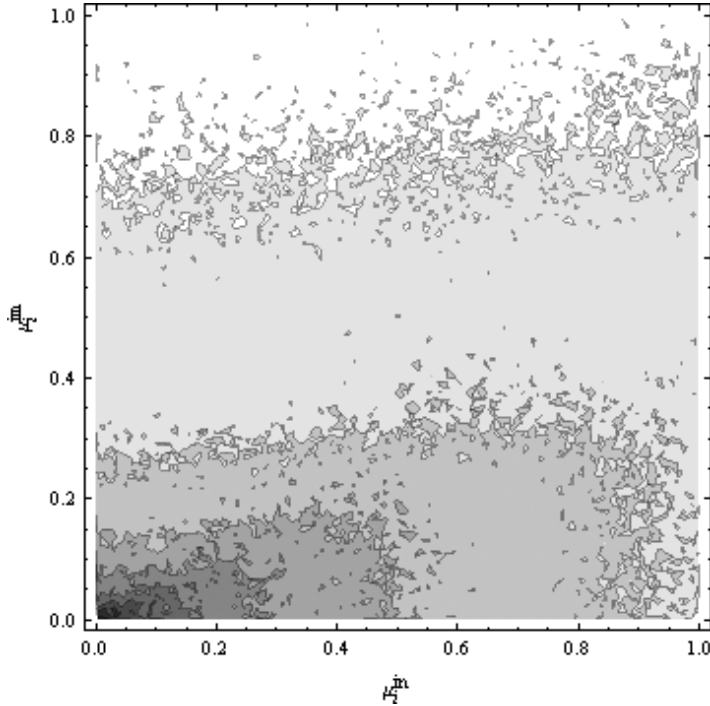


Fig. 4. Contour plot of the performance ( $\bar{\pi}_i$ ) surface ( $\sigma = 0.1; \rho = 1$ ).

is valuable only for those agents who have sufficiently strong comparative advantage in innovation over imitation. For those agents with comparative advantages in imitation, having a greater innovation capability can actually diminish their performance as they substitute away from imitation and toward innovation.<sup>j</sup>

**Property 2.** An agent's performance monotonically increases in his/her imitation skill, independently of his/her innovation skill. Conversely, for an agent with a given imitation skill, the impact of his/her innovation skill on the performance is non-monotonic; an increase in innovation skill can diminish the agent's performance if he/she has sufficiently poor innovation skill relative to imitation skill.

Figure 4 shows that the performance level is highest for those agents who have a high capability in imitation *and* a low capability in innovation. In fact, these agents appear to outperform those agents who are superior in *both* imitation and innovation. The agents who are highly capable in innovation but are deficient in

<sup>j</sup>This is the direct implication of innovation and imitation being substitutes at the individual level. The substitution of innovation for imitation by sufficiently innovative individuals generates benefits for other imitative individuals (at the expense of the originators of those ideas who are unable to capture the full benefits).

imitation perform relatively poorly. As expected, the worst performance is attained by the agents who are deficient in both individual and social learning.

This result suggests that an agent's performance is driven, in large part, by his/her ability to imitate and, hence, the structure of networks is important in the process of learning. Note that the act of imitation entails copying ideas that are being used by others; those ideas are a mix of ones that have been selected and ones that are random (ideas that agents were endowed with and have not had a chance to change or those that are effectively random because they were adopted long ago when the environment was very different). Innovation, on the other hand, entails random draws from the entire space of ideas. If the environment is relatively stable, the act of imitation should be more productive as the imitating agent's implemented ideas come from a favorably biased sample.<sup>k</sup> The performance differential between the agents who specialize in imitation and those who specialize in innovation should then be affected by the parameters of environmental turbulence,  $\sigma$  and  $\rho$ , which I find to be the case in Sec. 5.2.

In sum, my model generates interactive dynamics between learning mechanisms such that a small group of agents specializing in innovation supplies fresh discoveries for the entire population, thereby benefitting all who have access to these discoveries through their networks. Unfortunately for those agents skilled in innovation, however, it is the agents with superior skills in developing networks and copying others' ideas who capture the disproportionate share of the benefits. In the next section, I ask how the environmental factors such as  $\sigma$  and  $\rho$  affect these properties.

## 5.2. *The impact of turbulence in the task environment*

Interpreting and reporting the impact of  $\sigma$  and  $\rho$  on the entire population is a daunting task. For analytical and expositional simplicity, I select four special groups of agents and focus on their behaviors [see Fig. 5]:

$$\begin{aligned}
 S &\equiv \{\text{all } i \mid \mu_i^{in} \geq 0.8 \text{ and } \mu_i^{im} \geq 0.8\}; \\
 N &\equiv \{\text{all } i \mid \mu_i^{in} \geq 0.8 \text{ and } \mu_i^{im} < 0.2\}; \\
 M &\equiv \{\text{all } i \mid \mu_i^{in} < 0.2 \text{ and } \mu_i^{im} \geq 0.8\}; \\
 C &\equiv \{\text{all } i \mid \mu_i^{in} < 0.2 \text{ and } \mu_i^{im} < 0.2\}.
 \end{aligned}
 \tag{4}$$

$S$  denotes the set of *Super-agents* who are superior in both innovation and imitation.  $N$  is the set of *Innovators* who are very good at innovation but poor at imitation.  $M$  represents the group of *Imitators* who are very good at imitation but poor at

<sup>k</sup>Given this intuition, one may question why the entire population does not evolve to learning by imitation only. The fact is that such a system will become completely homogeneous in terms of ideas and, thus, will eventually run out of fresh ideas to be passed around. It is at this stage that innovation will then become relatively more attractive again. The findings on "specialization" as described in the previous section then imply that the steady-state of the social system involves co-existence of specialized learning mechanisms where a small group focuses on innovation and the rest of the population focuses on imitating the ideas generated by those innovators.



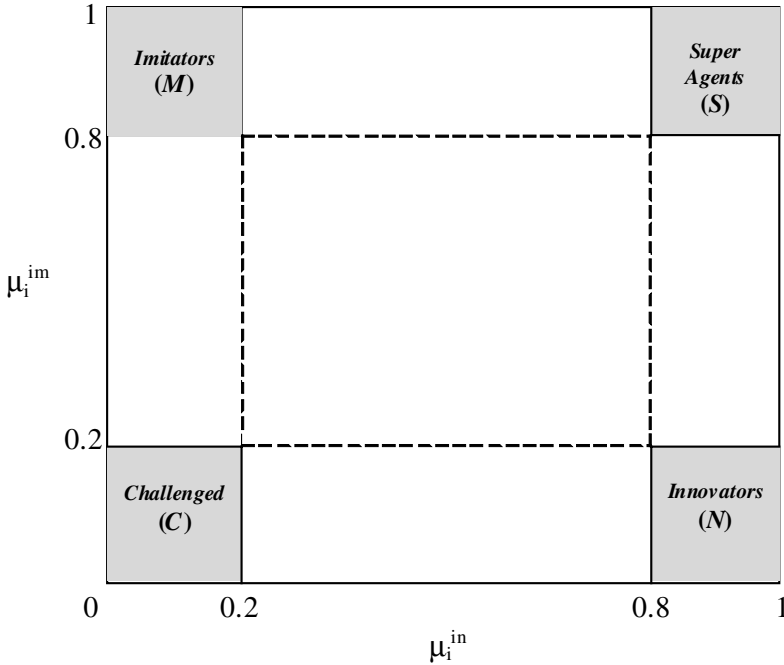


Fig. 5. Four special agent-types.

innovation. Finally,  $C$  is the group of *Challenged-agents* who have extremely limited skills in both innovation and imitation. This typology is useful for summarizing the comparative dynamics results for the meaningfully defined sub-populations.

Using the above typology, I first compute the steady-state values of the endogenous variables for each type as the simple averages over the values of all individuals belonging to the group:

$$\hat{q}^G = \frac{1}{|G|} \sum_{\forall i \in G} \bar{q}_i; \quad \hat{\pi}^G = \frac{1}{|G|} \sum_{\forall i \in G} \bar{\pi}_i, \quad (5)$$

where  $|G|$  is the size of the set  $G \in \{S, N, M, C\}$ . I compute  $\hat{q}^G$  and  $\hat{\pi}^G$  for each replication and then take their averages over the one hundred replications.

The impact of  $\sigma$  and  $\rho$  on these group-level steady-state values are reported in Fig. 6. The top figure plots the values of  $\hat{q}^G$  for all four types for  $\sigma \in \{0.1, 0.2, 0.3, 0.5\}$  given  $\rho = 1$ , while the bottom figure plots the same information for  $\rho \in \{1, 3, 5, 9\}$  given  $\sigma = 0.1$ .

The first thing to note is the dramatic divergence in the value of  $\hat{q}^G$  between the *Innovators* ( $N$ ) and the *Imitators* ( $M$ ). The *Innovators* specialize in innovation, while the *Imitators* specialize in imitation:  $\hat{q}^N \cong 1$  and  $\hat{q}^M \cong 0$  for all values of  $\sigma$  and  $\rho$ . Such division of cognitive labor was identified and discussed in the

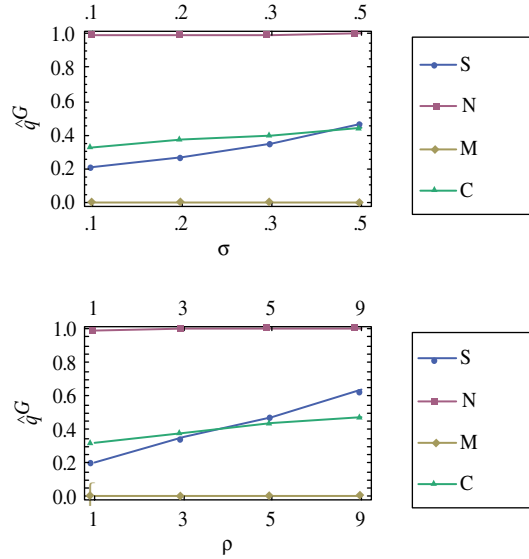


Fig. 6. Impact of environmental volatility on the mean innovation probability,  $\hat{q}^G$ , for each skill type.

previous section, but Fig. 6 shows that the property is robust to varying degree of environment turbulence.

Examining the specific numerical values for the points plotted in these two figures, I find that  $\hat{q}^N$ ,  $\hat{q}^S$ , and  $\hat{q}^C$  all increase in  $\sigma$  and  $\rho$ , while  $\hat{q}^M$  actually declines in  $\sigma$  and  $\rho$ .<sup>1</sup>

**Property 3.** The *Innovators*, *Super-agents*, and the *Challenged-agents* pursue innovation more intensely when they are in a more turbulent environment. The *Imitators* reduce their innovation intensity when in a more turbulent environment.

Note that imitation entails copying from another agent an idea that was originally adopted by that agent for an environment that existed in the past. More specifically, the set of ideas from which a random draw is made under imitation is biased toward being adaptive to the past environment, while the ideas available for innovation are taken from the entire space of ideas and, hence, are unbiased. For this reason, ideas copied from another agent tend to become obsolete at a faster rate when the environment is more turbulent. Consequently, a more turbulent environment raises the attractiveness of innovation relative to imitation for many agents. Given that the *Innovators* and *Super-agents* are both endowed with

<sup>1</sup>The numerical values of  $\hat{q}^M$  for various  $\sigma$  and  $\rho$  are as follows. For the top figure,  $\hat{q}^M$  takes the values of  $\{0.00357198, 0.00247447, 0.00192826, 0.00186038\}$  for  $\sigma \in \{0.1, 0.2, 0.3, 0.5\}$ , respectively. For the bottom figure, it takes the values of  $\{0.00357198, 0.00204249, 0.00177487, 0.00172952\}$  for  $\rho \in \{1, 3, 5, 9\}$ , respectively.

superior innovation skills, their response to the increased environmental turbulence is to raise their rates of innovation. [The *Challenged-agents* also pursue greater rate of innovation as the driver of their endogenous choice is the *relative* strength of their endowed learning skills.] Given the inherent mechanism of division of cognitive labor identified previously, the *Imitators* then utilize their superior imitation skills and take advantage of the increased innovation activities of other types of agents as the environment becomes more turbulent and unstable. This results in a lower rate of innovation for the *Imitators* and higher rates of innovation for other types of agents when the environment is more turbulent.

Contrary to the *Innovators* and *Imitators* who pursue extreme specializations, *Super-agents* and *Challenged-agents* pursue innovation with moderate levels of intensity. In a relatively stable environment where the common goal does not shift very much from one period to the next, *Challenged-agents* tend to engage in innovation more than *Super-agents*. *Super-agents* find it more beneficial to utilize their superior imitation capabilities in a relatively stable environment and refrain from engaging in innovation. However, when the environment is highly turbulent, *Super-agents* find it necessary and desirable to substitute imitation with innovation as they are endowed with superior capacity to innovate. *Challenged-agents*, with their rather limited ability to innovate, are not able to respond to the turbulent environment with such flexibility.

A final observation is that *Super-agents* who are as good as *Innovators* in terms of their innovation capabilities end up devoting less time to innovating than *Innovators*. Clearly, what matters is then the *relative* abilities rather than the *absolute* abilities. *Super-agents* who are capable in both innovation and imitation tend to shift their focus away from innovation and more toward imitation (which is privately more rewarding as we saw in Sec. 5.1).

Next we examine the impact of  $\sigma$  and  $\rho$  on the performance of agents with heterogeneous learning skills. Figure 7 plots  $\hat{\pi}^G$  for the same sets of  $(\sigma, \rho)$  configurations as above. It is clear that  $\hat{\pi}^G$  is lower for agents of all types in a more turbulent environment. A more turbulent environment tends to throw the agents further away from the ever-changing optimum. Since a greater distance to the organizational goal implies poorer performance, it follows that an increase in environmental turbulence leads to deteriorating performance for all types of agents.

**Property 4.** For agents of all skill types, performance is lower when the task environment is more turbulent.

In terms of the type-specific performance levels, it is highest for *Imitators*, followed by *Super-agents*. *Innovators*, while ahead of *Challenged-agents*, fall short of *Super-agents* and *Imitators*. The group-level results here are consistent with the agent-level results captured in Fig. 4. It is clear that an agent's performance is strongly affected by his/her imitation capability,  $\mu_i^{im}$ ; more so than the innovation capability,  $\mu_i^i$ .

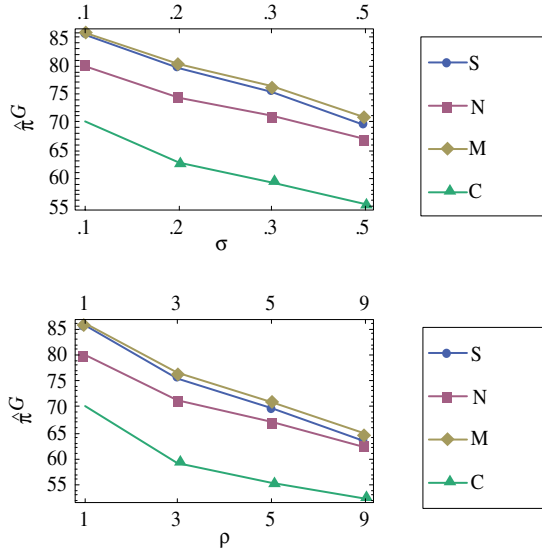


Fig. 7. Impact of environmental volatility on the mean performance of each skill type.

Finally, it is worth noting that the performance differential between the *Imitators* and the *Innovators*,  $\hat{\pi}^M - \hat{\pi}^N$ , decreases in  $\sigma$  and  $\rho$ .<sup>m</sup> This supports the earlier argument that the relative benefits from the act of imitation over that of innovation decreases in the degree of turbulence, because the ideas of others are more likely to be obsolete in a more turbulent environment.

### 6. Steady-State Structure of the Endogenous Social Learning Network

Given the prominent roles that the imitation capability,  $\mu_i^{im}$ , and the intensity of imitation activity,  $1 - \bar{q}_i$ , play in determining an agent’s performance, I now probe deeper into the social learning process by investigating the exact structure of the social learning networks that develop within the organization.

The raw materials we have in hand for inferring the network structure of agent  $i$  are the steady-state observation probabilities,  $\{\bar{p}_i^j\}_{\forall j \neq i}$ . Note that  $\bar{p}_i^j$  represents the probability with which agent  $i$  observes another agent  $j$  in steady-state, given that he chooses to engage in social learning (imitation) rather than individual learning (innovation). For each of the 100 independent replications and for each agent  $i$  (from the population of 150 agents) we have 149  $\bar{p}_i^j$ ’s, one for each agent  $j \neq i$ .

There are two ways in which one can study the endogenous architecture of the networks in this setting. One approach is to actually create directed links between

<sup>m</sup>To be specific,  $\hat{\pi}^M - \hat{\pi}^N$  takes the values of  $\{5.99, 5.91, 5.28, 4.01\}$  for  $\sigma$  values of  $\{0.1, 0.2, 0.3, 0.5\}$ , respectively. Likewise,  $\hat{\pi}^M - \hat{\pi}^N$  takes the values of  $\{5.99, 5.30, 4.08, 2.51\}$  for  $\rho$  values of  $\{1, 3, 5, 9\}$ , respectively.

the agents using the above steady-state imitation probabilities, thereby generating the steady-state networks of social learning which can then be studied using standard analytical tools in the network literature. Alternatively, one could stay strictly within the confines of the current model and study the distributions of these probabilities directly, exploring the impact the turbulence parameters have on the distributions. I take the latter approach in this paper.<sup>n</sup>

### 6.1. *Selectivity of individual networks*

Note that in my model, each agent has his/her specific learning network that is based on the likelihood of his/her observing each of the other agents in the population. I start my analysis by first examining how the individual agent's learning skills affect the general structure of the network he or she ends up with in the steady-state. There are two things I am interested in: (1) how selective is an individual's network and (2) how central is an individual in other agents' networks. I start with the network selectivity.

In addressing the selectivity of a network, I ask the following question. Does an agent learn from a small number of other agents or does he learn from many? In our context, this question can be addressed by observing the distribution of  $p_i^j(t)$ 's: Does he have a sharply defined network in which he observes a small subset of other agents with high probabilities or does he choose his target agent randomly from the population? If an individual is equally likely to imitate other agents in the population — so that  $p_i^j(t) = 1/(L-1)$  — there is no selectivity in the agent's network as imitation is completely (that is, uniformly) random. Alternatively, if the probability of observing another agent is concentrated on a single individual — so that  $p_i^j(t) = 1$  for some  $j$  — then there is a maximal selectivity in the agent's network.

An appropriate measure for this purpose entails [11] “entropy” which was originally defined in the context of information theory as an inverse measure of the information content of a message. In adapting this measure for the context at hand, the entropy measure for the network of agent  $i$  is defined to be:

$$E_i(t) \equiv - \sum_{\forall j \neq i} p_i^j(t) \cdot \log_2 p_i^j(t). \quad (6)$$

The “selectivity” of agent  $i$ 's network in time  $t$  is then

$$s_i(t) \equiv E_{\max} - E_i(t), \quad (7)$$

where  $E_{\max}$  is the maximum entropy value which is equal to  $\log_2(L-1)$ . The steady-state value of the agent  $i$ 's network selectivity is  $\bar{s}_i = \frac{1}{5,000} \sum_{t=10,001}^{15,000} s_i(t)$ .

<sup>n</sup>The first approach was taken in an earlier version of the paper. At the suggestion of a referee, I have chosen to explore the second approach for the final version. I have confirmed that both approaches generate results that are fully consistent with each other and all of the qualitative results obtained in the earlier version remain valid in the current version.

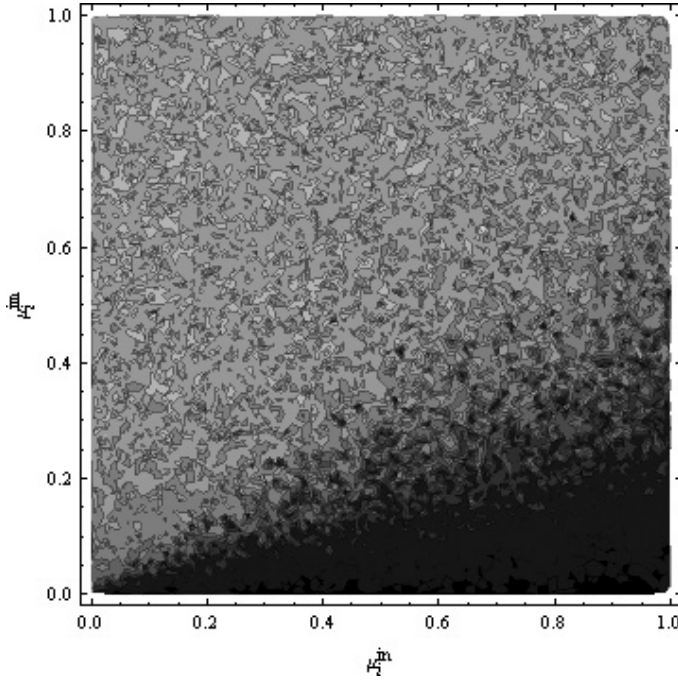


Fig. 8. Selectivity ( $\bar{s}_i$ ) of individual agent's network ( $\sigma = 0.1; \rho = 1$ ).

An agent's steady-state network, hence, becomes more (less) selective as  $\bar{s}_i$  increases (decreases).

To start, I compute the values of  $\bar{s}_i$  for all  $i$  in the population with varying learning skills. The contour plot of the selectivity surface, generated for the baseline parameter values of  $\sigma = 0.1$  and  $\rho = 1$ , is provided in Fig. 8. The lighter (darker) the gray-level, the higher (lower) is the selectivity of an agent's network. Notice that the selectivity surface is almost a mirror-image of the innovation choice ( $\bar{q}_i$ ) surface displayed in Fig. 2. Hence, it is those agents who pursue imitation with greater intensity (lower values of  $\bar{q}_i$ ) who end up having networks that are strongly selective (higher values of  $\bar{s}_i$ ). Generally, an increase in  $\mu_i^{im}$  monotonically raises the level of selectivity, while an increase in  $\mu_i^{in}$  reduces the level of selectivity.

**Property 5.** The selectivity of an individual learning network increases in the agent's imitation skill and decreases in the innovation skill.

How is the selectivity of a network affected by the frequency and the variability of the task environment? While it is possible to present the contour plots for various combinations of  $\sigma$  and  $\rho$ , it is difficult to identify the general pattern given the widely varying impacts the parameters have on the agents with heterogeneous learning skills. As such, I focus on the previously defined four special groups of agents,

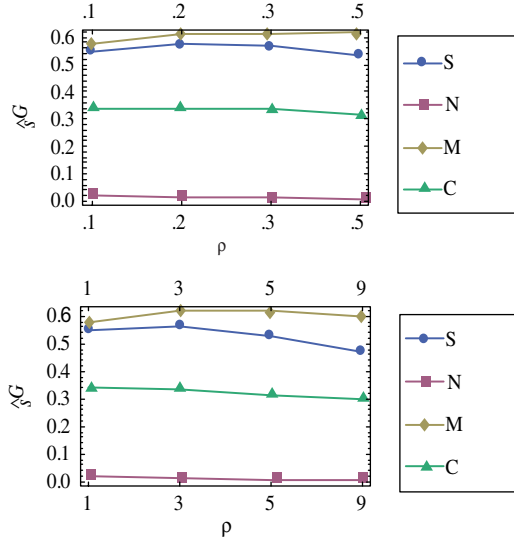


Fig. 9. Impact of environmental volatility on the mean selectivity of networks within types.

$\{S, N, M, C\}$ . Denote by  $\hat{s}^G$  the group-level average of the network selectivity such that

$$\hat{s}^G = \frac{1}{|G|} \sum_{\forall i \in G} \bar{s}_i, \quad (8)$$

where  $|G|$  is the size of the set  $G \in \{S, N, M, C\}$ . I compute  $\hat{s}^G$  for each replication and then take their averages over the one hundred replications. The impact of  $\sigma$  and  $\rho$  on these group-level steady-state values are reported in Fig. 9. The top figure plots the values of  $\hat{s}^G$  for all four types for  $\sigma \in \{0.1, 0.2, 0.3, 0.5\}$  given  $\rho = 1$ , while the bottom figure plots the same information for  $\rho \in \{1, 3, 5, 9\}$  given  $\sigma = 0.1$ .

First of all, the *Imitators* have the most selective network, followed by the *Super-agents*, the *Challenged-agents*, and the *Innovators* in decreasing order. It is worthwhile to note that the networks held by the *Innovators* are essentially random. Given the intensity with which they pursue innovation, they simply do not have time to build and refine their social learning networks.

The turbulence parameters,  $\sigma$  and  $\rho$ , have differential impact on these agent-types. For both *Challenged-agents* and the *Innovators*, an increase in  $\sigma$  or  $\rho$  monotonically reduces the selectivity of their networks. This is simply due to the fact that these two types rely heavily on their own innovation (because they both have severely limited imitation capability) and they tend to step up their innovation efforts when the task environment is more turbulent. On the other hand, the *Imitators* and *Super-agents* — both with superior imitation capabilities — tend to raise the selectivity of their networks in a more turbulent task environment as long as the degree of turbulence is not too high. They actually find it worthwhile to utilize

their networks to a greater extent and take advantage of the increased innovation activities of the *Innovators* and the *Challenged-agents*. This network advantage is of limited use, however, if  $\sigma$  or  $\rho$  is too high; the impact of turbulence on the selectivity is non-monotonic for the *Innovators* and the *Super-agents*.

## 6.2. Centrality of individual agents

The next structural measure of the networks I examine is the overall *centrality* of an agent in the system of interacting networks. In the standard network literature, this is often called the *prestige* of an agent and is measured by the number of links that are directed toward the node representing the agent. In our context, this is captured by the probability with which a given agent is observed by other agents in the population. More precisely, denote by  $c_i(t)$  the centrality of agent  $i$  within the population:

$$c_i(t) \equiv \sum_{\forall j \neq i} (1 - q_j(t)) \cdot p_j^i(t). \quad (9)$$

In order for agent  $i$  to be observed by agent  $j$ , agent  $j$  must first connect to his network, which happens with probability  $(1 - q_j(t))$ , and then select agent  $i$  with the probability of  $p_j^i(t)$ . The probability that agent  $i$  is observed by agent  $j$  is then  $(1 - q_j(t)) \cdot p_j^i(t)$ . The probability that agent  $i$  will be observed by *any* agent in the population is then the sum of those probabilities over the entire population (other than  $i$  itself).

Again, I focus on the steady-state by computing the value of agent  $i$ 's centrality averaged over the periods between  $t = 10,001$  and  $15,000$ :  $\bar{c}_i = \frac{1}{5,000} \sum_{t=10,001}^{15,000} c_i(t)$ . An agent becomes more (less) central as  $\bar{c}_i$  increases (decreases). Figure 10 captures the contour plot of the centrality surface as a function of the learning skills.

Two things are immediate from Fig. 10. The overall structure of the centrality surface is somewhat similar to that of the performance surface as captured in Fig. 4. Hence, those agents with higher centrality scores tend to perform better than those with lower centrality scores. In fact, for those 15,000 agents considered in this baseline study, the correlation between an agent's centrality and his/her performance was 0.7. This is consistent with the available empirical findings that show a positive relationship between centrality and performance [12].

**Property 6.** The performance of an agent is positively related to his/her centrality.

Secondly and relatedly, it appears that the agents in the upper left region ( $M$ ) of the skills space tend to be more central than the ones in the upper right ( $S$ ) or lower right regions ( $N$ ) — and certainly much more so than the agents in the lower left corner ( $C$ ). This is precisely captured in Fig. 11, in which the group-level averages,  $\hat{c}^G$ , are plotted for different values of turbulence parameters,  $\sigma$  and  $\rho$ . It is clear that  $\hat{c}^M > \hat{c}^S > \hat{c}^N > \hat{c}^C$  for all  $\sigma \in \{0.1, 0.2, 0.3, 0.5\}$  and  $\rho \in \{1, 3, 5, 9\}$ . This study then provides a potential explanation for the positive relationship between



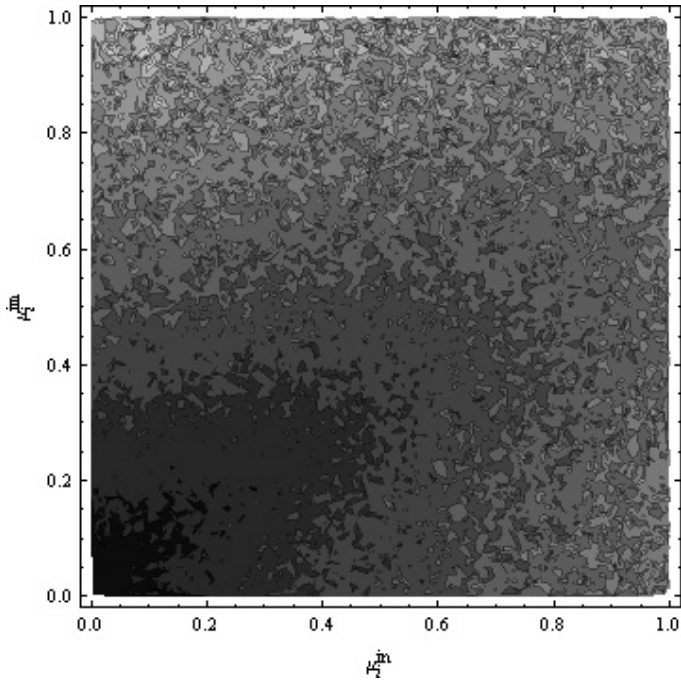


Fig. 10. Centrality ( $\bar{c}_i$ ) of individual agents ( $\sigma = 0.1; \rho = 1$ ).

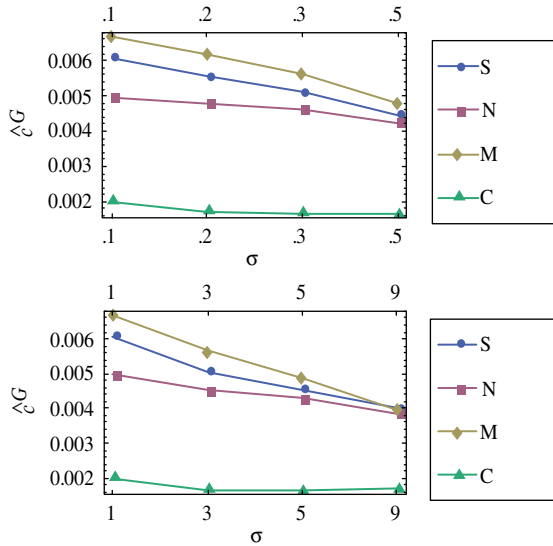


Fig. 11. Impact of environmental volatility on the mean centrality of agents within types.

centrality and performance: The ability to engage in social learning (imitation) enables an agent to combine diffused knowledge in the population and attain superior performance for itself. But by also acting as the repository of useful ideas for other agents in the population, these agents with superior imitation skills become central to other agents' learning networks in the long run.

Finally, the degree of centrality is generally reduced in a more turbulent environment for all types of agents. This is consistent with the intuition that innovation tends to be relatively more effective in a more turbulent environment because the ideas copied from others are less likely to be adaptive in changing environments. As the agents are more likely to innovate than imitate, the probability of any given agent being observed by another agent tends to be low.

### 6.3. *Endogenous flow of knowledge*

While the centralities of the agents with heterogeneous learning skills give us some notion of how important is a particular learning skill in the overall social learning network, it does not tell us who learns from whom. In this section, I explore the direction in which knowledge flows in the system of interacting networks.

Note that both  $\mu_i^{in}$  and  $\mu_i^{im}$  can take values from the unit interval of  $[0, 1]$ . Let us divide the unit interval into five equal-sized sub-intervals —  $[0, 0.2)$ ,  $[0.2, 0.4)$ ,  $[0.4, 0.6)$ ,  $[0.6, 0.8)$ , and  $[0.8, 1.0]$ . Using this sub-divisions, we can then divide the space of learning skills into 25 equal-sized regions based on the values of  $\mu_i^{in}$  and  $\mu_i^{im}$ . The 150 agents created in each replication will be scattered across these regions based on their endowed learning skills. I focus on the flow of information from the population to each of the four special groups,  $\{S, N, M, C\}$ , and use this information to identify the learning relationships between these groups.

Let  $O_T$  be the set of agents with endowed learning skills that fall within a given target region (out of the 25 regions in the skills space). The probability that a given agent  $i$  in  $G \in \{S, N, M, C\}$  will observe an agent in  $O_T$  is the sum of the probabilities that he will observe each agent in  $O_T$  — i.e.  $\sum_{\forall j \in O_T} \bar{p}_i^j$ . The probability that *an* agent in  $G$  will observe an agent in  $O_T$  is then the average of these aggregate probabilities over all agents in group  $G$ :

$$\frac{1}{|G|} \sum_{\forall i \in G} \sum_{\forall j \in O_T} \bar{p}_i^j. \quad (10)$$

For each  $G \in \{S, N, M, C\}$ , I compute these probabilities for all 25 target groups,  $O_T$ . Figure 12 captures these probabilities for the three groups,  $M$  (top),  $S$  (middle), and  $C$  (bottom), with the gray level of a given cell representing the size of the probability — i.e. the lighter the gray level, the higher is the probability. I omit the figure for the  $N$  group, because an *Innovator's* network is close to random and thus not very informative.

While these figures show the probabilities of observing all 25 groups, I focus only on the flow of knowledge among the four special groups. The top figure shows

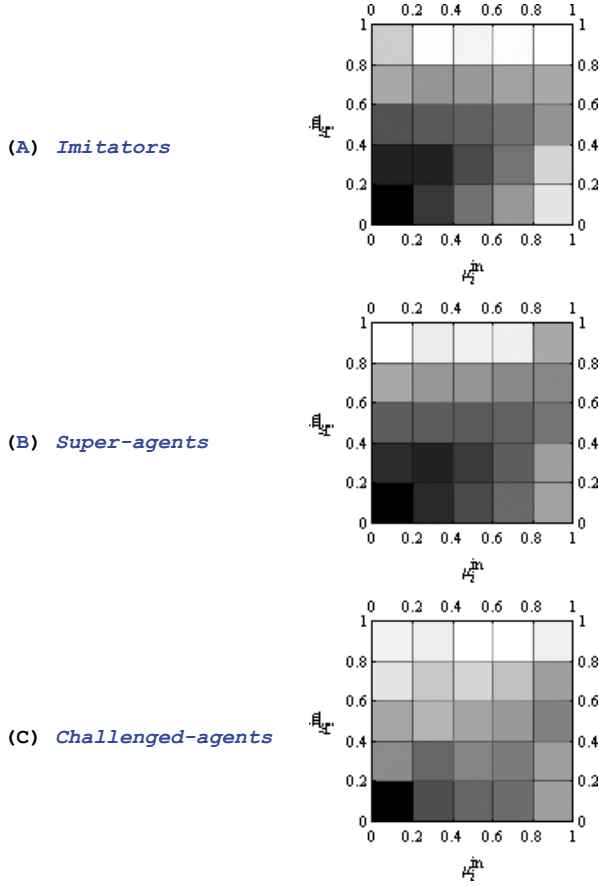


Fig. 12. Mean probability of an agent in each type ( $M$ ,  $S$ ,  $N$ ) observing an agent in other groups ( $\sigma = 0.1$ ;  $\rho = 1$ ).

the mean probability of an *Imitator* ( $M$ ) observing an agent in each of the 25 sub-groups (cells). It is clear that the *Imitators* ( $M$ ) learn from the *Super-agents* ( $S$ ) most intensively, and then from the *Innovators* ( $N$ ). The middle figure shows the mean probability of a *Super-agent* ( $S$ ) observing an agent in each of the sub-groups. The *Super-agents* ( $S$ ) learn most intensively from the *Imitators* ( $M$ ), and then from the *Innovators* ( $N$ ). Finally, the bottom figure shows the same information for a *Challenged-agent* ( $C$ ). The *Challenged-agents* ( $C$ ) learns most intensively from both the *Imitators* ( $M$ ) and the *Super-agents* ( $S$ ), and then from the *Innovators* ( $N$ ).

The property of the social learning network that evolves can be summarized in a flow diagram of knowledge among the four special types. This is shown in Fig. 13. The two diagrams display the direction of knowledge flow using arrows from one group to another. The top diagram is for  $\sigma = 0.1$  and the bottom diagram is for  $\sigma = 0.5$ . To capture the essential structure of the knowledge network, I

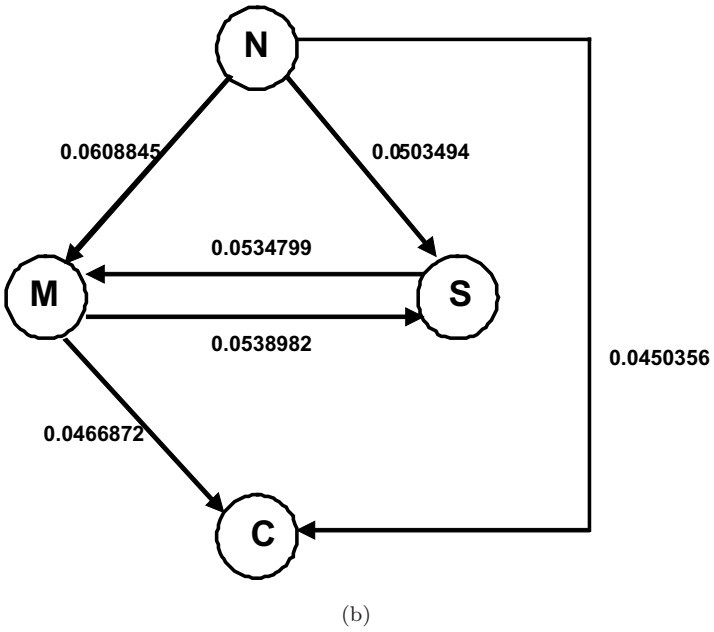
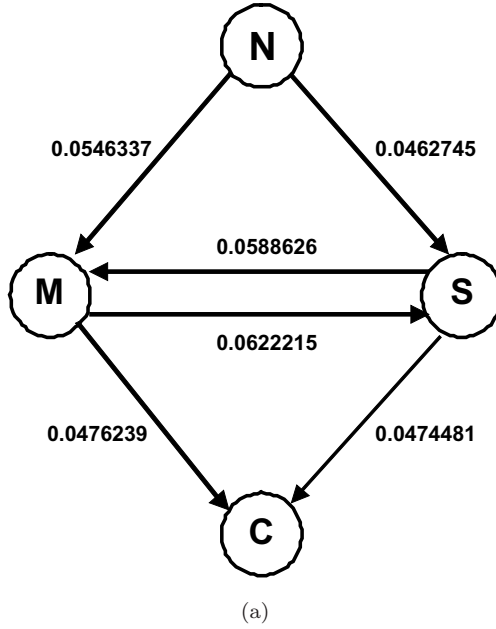


Fig. 13. Flow of knowledge for (a)  $\sigma = 0.1$  and (b)  $\sigma = 0.5$ .

have captured only those links that have probabilities above 0.045.<sup>o</sup> The actual probabilities are shown for each link that represents the specific flow of knowledge. First of all, notice that the *Innovators* are the main source of knowledge. The ideas generated by the *Innovator* are picked up by the *Imitators* and the *Super-agents* who then pass them on to the *Challenged-agents*. Rather than learning directly from the *Innovators*, the *Challenged-agents* tend to learn second-hand from the *Imitators* and the *Super-agents*. It should also be noted that there exist a substantial amount of mutual learning by the *Imitators* and the *Super-agents*.

**Property 7.** Both *Imitators* and *Super-agents* play the role of a connector in the network. Knowledge then flows from the *Innovators* to the two connectors, and then to the *Challenged-agents* through the connectors. The two connectors engage in a significant amount of mutual learning among themselves.

To see the impact of the environmental turbulence, compare the two diagrams. In going from  $\sigma = 0.1$  to  $\sigma = 0.5$ , the mutual learning between the *Imitators* and the *Super-agents* is diminished, but these two groups of agents raise the intensity of direct learning from the *Innovators*. The *Challenged-agents* also reduce their learning from the connectors — the *Imitators* and the *Super-agents* — and instead learn directly from the *Innovators*. In fact, the probability of their learning from the *Super-agents* fall below the threshold of 0.045 and the link from the *Super-agents* to the *Challenged-agents* is no longer displayed in the figure, while a new link appears from the *Innovators* to the *Challenged-agents*.

## 7. Concluding Remarks

There are two ways in which this paper contributed to the literature on social and organizational learning. First, I developed a formal model of social learning which enabled me to *evolve* social networks and characterize their emergent structures. In the process, I explored the extent to which the roles individuals come to play within a social/organizational system are determined by the innate skills they have as well as the characteristics of the surrounding environment. Second, my modeling approach made a methodological contribution to the literature on social network analysis. As described in detail, the process of social learning was modelled in the context of evolving networks having *probabilistic ties*. This feature of the model distinguishes it from the existing models of social networks which treat the links between individuals as being deterministic.

Several interesting findings emerged from this study. I will only mention a few in concluding this paper. *Super-agents* who are as skilled in innovation as *Innovators* devote less time to innovating than *Innovators*. Given their superior skills in both

<sup>o</sup>Given that there are 150 agents per replication, a uniformly random network will give us a probability of 0.0067 (=1/149) that an agent will observe another agent in the population. Since there are six agents per sub-group on average, an agent with a uniformly random network will observe an agent in a given sub-group with the probability of 0.04.

innovation and imitation, they find it privately beneficial to focus more on imitation. Having the extra skill for imitation means *super-agents* tend to outperform *Innovators*. However, I also find that *Super-agents* are outperformed by *Imitators* who are skilled in imitation but relatively poor at innovation. Possessing superior skills in both innovation and imitation turns out to be a double-edged sword for *Super-agents*: Even though their strong imitation skill gives them an advantage over *Innovators*, their equally strong innovative skill induces them to pursue innovation to an excessive degree such that they end up generating (unintended) benefits to *Imitators* while incurring the implicit cost for themselves in the form of foregone values from imitating others.

My modeling approach also allowed detailed examinations of the micro structure of the emergent social learning networks. In particular, the selectivity of an individual learning network was found to increase in the agent's imitation skill and decrease in the innovation skill. I was also able to explore the impacts that endowed learning skills have on the centrality of the individuals. An important result is that the highest centrality goes to *Imitators* who are superior in imitation but poor in innovation. By playing the role of connectors in the emergent network they prove essential to the efficient diffusion of knowledge in the organization. It is significant that the network centrality held by *Imitators* surpasses not only that of *Innovators* but also of *Super-agents*. It should be noted that this result is fully in line with the available empirical findings [12].

The model presented here focused on the social learning network in a single organization. For that organization, the population of agents was also held fixed for the entire horizon under consideration. There are two ways in which the model can be enriched. First, social learning by individual agents can take place in a bigger social system containing multiple organizations. In this framework, an individual can then learn from another agent in the same organization or from an agent in another organization. An important factor to be considered in this setting is the (highly likely) possibility that the organizations may have different goals and, hence, the comparative value of internal versus external social learning is likely to depend on the scale and scope of the inter-organizational heterogeneities. Second, the agents are likely to be mobile in real organizations. New agents may join an organization, while some existing agents may exit it. Modelling and exploring the endogenous transformation of the social learning network when the identities of the agents (nodes) are constantly changing is a highly challenging task that is left for future research.

## Acknowledgments

I appreciate the comments of the two referees and the participants at the 2010 International Workshop on Emergent Intelligence on Networked Agents (WEIN) — Toronto, CA. This work was supported by the College of Graduate Studies at Cleveland State University.

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